

Survey of Occupant Load Densities in Retail Buildings

Report By:

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Forward:

Occupant load is a critical factor in egress analyses for fire safety systems. Codified values for occupant load for many occupancies are estimates which frequently do not reflect actual conditions. Further, a full statistical analysis of the data, taking into account its variability, is often not undertaken and reported. For retail buildings in particular, the variability of the occupant load can provide important design information.

The project described in this report provides:

- an in-depth assessment of currently available people counting systems, identifying strengths and weaknesses.
- a review of occupant loads in actual fire incidents in retail buildings to assess optimum measurement intervals and statistical analyses.
- an in-depth statistical analysis of occupant data in Swiss retail stores, exploring the impact of occupancy detailed type, building configuration and location.

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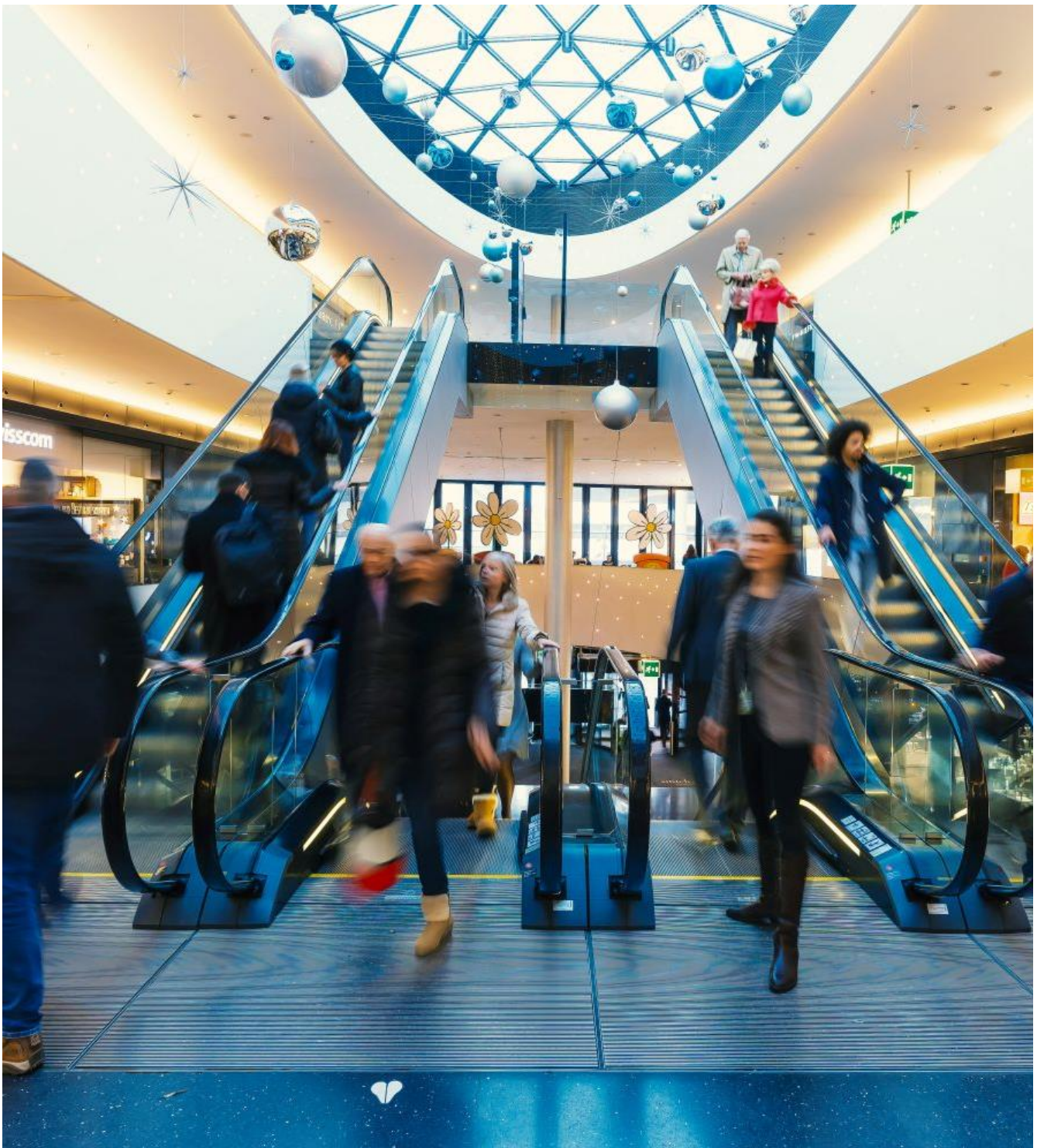
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The Society of Fire Protection Engineers (SFPE) established its Educational and Scientific Foundation in 1979. The Foundation is a charitable 501(c)(3) organization incorporated in the United States of America and supports a variety of research and education programs to foster its mission to is to enhance the scientific understanding of fire and its interaction with the natural and built environment.

Keywords: human behavior, retail buildings, occupant loads, people counting systems

Survey of occupant load densities in retail stores

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Abstract

The occupant load density is an important parameter for evaluating evacuation safety problems and designing the means of egress. Occupant load density values for retail buildings are typically found in fire safety guidelines or standards, though, it is unclear on which data or estimates these are based. Some values can be traced back more than 60 years. In addition, studies have shown that these values are not realistic. Therefore, there is a clear need for systematic data collection and formulation of best practices regarding what occupant load densities to use. In this report we discuss how to assess occupant load density as well as which measurement systems of occupant load density can be applied.

In order to assess occupant density in Switzerland, we conducted a wide-scale survey looking at the density of persons in retail buildings over the period of one year. 96 stores with different retail types participated, including supermarkets, malls, department stores, electronic shops, hardware shops, clothing shops, furniture shops, and sports shops. The participating stores were drawn from the basic population of Swiss stores to form a representative sample and the measurements taken using a set of specific requirements. For supermarkets the influence of the sales area and the location – especially for supermarkets located in highly frequented areas – was also investigated.

The results of this survey are presented in this report. The study shows that the values in the fire protection standards and guidelines are typically very high and do not provide a realistic basis for determining occupant load density. In addition, we found the retail type to be a decisive factor for determining occupant load density. The current differentiation by building floor level present in many fire safety codes should be questioned, since the data shows no big differences between the floor levels.

Furthermore, the data shows a high variability in occupant load density and the distribution is heavily right-skewed. This is a challenge for performance-based design, where often a worst-credible case is used to justify design decisions. Additionally, an appropriate consideration of the distribution of the occupant load density would require a probabilistic or risk-based analysis. In order to allow for such analyses, we provide data for different quantile values of the occupant load density and other sample statistics as well as parameters of fitted probability distributions for different retail types.

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1. Introduction

The risk of human losses in fires can be reduced by an accurate design of the means of egress. The design of the means of egress and the proof of a safe evacuation of the occupants by an egress analysis implies the use of a reasonable estimation of the number of occupants to be evacuated. Therefore, the occupant load is a crucial parameter to ensure safety in buildings, with numerous surveys having been conducted in the past for different building uses. Most of the studies are based on walk-through investigation or assessment of building capacity, e.g. in residential buildings, hotels, hospitals, school buildings, theatres, cinemas, etc.

In contrast to this, the number of persons in retail stores is usually not influenced by the occupant capacity of a store. The number of persons is however highly influenced by individual choice and the current demand for store products. This leads to a high temporal variability of the number of retail store occupants. Therefore, walk-through investigations are not suitable for surveying retail buildings. A common approach is to assess the occupant load density during (supposed) peak sales days of the year. Then, the derived occupant load density is used as a design value. The drawback of this approach is that the variability in occupant load density remains unknown, since only one point in time, the one with a presumed maximum load, is assessed. Finally, it remains unclear whether such a design value is appropriate for representing the occupant load of retail occupancies during the rest of the year, since fires do not necessarily occur on-peak. Therefore, it is inadvisable to use this assessed value for the occupant load in probabilistic approaches used for risk assessment. Only survey methods that include the number of persons in time can be used for a proper survey of the occupant load density. To the authors knowledge, no publicly available survey has been systematically performed to date for retail occupancies. This report intends to provide:

- A systematic survey methodology to assess the occupant load densities in retail buildings,
- A statistical analysis of data from a survey of Swiss retail stores that was used to derive occupant load densities.

The findings of this report are part of a larger project in Switzerland aiming at a revision of the prescriptive requirements for the means of egress in Swiss retail buildings. The study was initiated by Espace.mobilité which is an interest group made up of leading Swiss retail companies. Its members are competitors on the market, but partners in more fundamental issues such as spatial planning, environmental protection, mobility, and building regulations. The study consists of three parts:

- Part 1: Review of normative specification on the occupant load and statistical analysis of fire events in retail buildings.
- Part 2: Survey of the occupant load density in Swiss retail buildings and determination of tentative design values.
- Part 3: Risk-based verification of the influence of the proposed design values on the means of egress by engineering methods.

A schematic overview of the structure of the project is shown in Figure 1.







Structure of the study "Survey of the occupant load density and verification of the design of the mean of egress for Swiss retail buildings".			
Part 1	I Introduction		Introduction and structure of the study
	II Fundamentals		Definitions, overview of normative specifications and analysis of fire events
Part 2	III Survey of the occupant load density in retail stores		Assessment method, measurement systems and dealing with measurement errors
	IV Occupant load density in Swiss retail stores		Selection of the sample, measurement concept and statistical evaluation of the data
	V Remarks on the determination of design values		Discussion on the determination of design values for design purposes
Part 3	VI Verification of the prescriptive design format for the mean of egress for retail buildings		Risk-based verification of the influence of the proposed design values on the means of egress by engineering methods

Figure 1: Outline of the project.

This SFPE report is limited to the findings of Parts 1 and 2 of the project. Part 3 deals with the influence of the determination of a design value on the determination of prescriptive egress requirements. Since the occupant load density in prescriptive regulations is used to directly define the egress requirements, an adjustment of this value will also lead to an adjustment of these requirements as well as a changed safety level compared to the current design. Therefore, it is important to quantify this level of safety and to assess the influence of a more realistic design value on the former before changing the occupant load factor in prescriptive codes.

2. Fundamentals

2.1 Definitions

Occupant load density

The occupant load density [persons/m²] in retail stores is defined as the occupancy [persons] of a fire compartment averaged by its gross fire compartment area [m²]. The occupancy of an area comprises all rooms accessible to the customer. Importantly it also includes streets and other traffic areas, apart from lavatory facilities with direct access to horizontal or vertical escape routes.

Retail stores

All sales areas in which goods or services are commercially offered for sale and which are accessible to the public.

Food retail stores

This includes retail stores with a majority of sales in the following categories:

- Fresh products (dairy products, bread, fruit/vegetables/salads, meat/poultry/fish, fresh chilled products)
- Drinks (with and without alcohol)
- Long-life foods (canned foods, confectionery, frozen foods, staple foods, cooking ingredients)

Near-food retail stores

This includes retail stores with a majority of sales in the following categories:

- Personal Care (personal care, paper and hygiene products)
- Animal needs

Non-food retail stores (specialist markets)

This includes retail stores with a majority of sales in the following categories:

- DIY stores (do-it-yourself/garden/car accessories)
- Home electronics (consumer electronics, household appliances, sound carriers, telecommunications, IT, photography)
- Fashion/Style (ladies/men/children/baby incl. sportswear, shoes incl. sports shoes, fashion, accessories, costumes, jewellery)
- Leisure (sporting goods and bicycles, luggage, leather goods, toys)
- Household/Living (furniture, home textiles, furnishings, household goods, seasonal decoration)

Supermarkets

Sales areas with a majority of sales consisting of food articles for daily use but can also have near-food and non-food articles in the assortment.

Highly frequented supermarkets

This includes supermarkets at important public transport terminals¹ with an authorised Sunday sale². As a guide, a total number of daily visitors of more than 7 persons per square metre of gross floor sales area can be expected in Switzerland³.

2.2 Normative specifications for the occupant load density

In general, the occupant load density depends on the size, use, and location of a building's rooms and is specified in fire safety guidelines or standards. In Switzerland, the occupant load density for retail buildings varies according to floor level, for example. It is possible to deviate from these values, but often only when compulsory details can be provided (e.g. seating plans for restaurants) or by providing measurements of the occupant load. This is especially difficult in the case of retail stores, as ongoing frequency measurements rarely exist, and no measurements are available in the case of new buildings. If no data on the occupant load density is available, the fire protection engineers are dependent on the assessment and approval of the fire protection authorities when deviating occupant load densities are proposed. As there is a lack of data on occupant load density, there are major differences in its subjective by different authorities. In some cases, due to the lack of data, the fire protection engineers are instructed to use the occupant load values from the prescriptive codes within a performance-based design. These values are seldomly well documented, as a recent review of codified occupant load factors by Spearpoint & Hopkin [1] concluded:

“The authors have not been able to source the original data used to select the occupant load factors, and therefore have been unable to identify the basis for these factors and to what distribution percentile they correspond. The difficulty in obtaining this information can make it hard for designers to determine the relevance of occupant load factors when applied to modern design and why there can be large disparities between jurisdictions. Whether these variations are solely down to cultural differences is not clear but it would seem the basis of current occupant load factors have origins that go back more than 60 years. By performing this review, a need to revisit mercantile floor space factors for new data has been highlighted.”

Spearpoint & Hopkin [1] reviewed different normative specifications for the occupant load density in retail stores in different guidelines (see Figure 2). They found that occupant load densities are interpreted very differently, densities for the main floor of retail stores varying between 0.1 pers./m² and 0.5 pers./m². To some extent the differences may be explained by cultural differences, but it is more likely that there are discrepancies in jurisdictional practices.

What is remarkable is the different interpretation of occupant load density regarding a building's floor levels. Some directives (e.g. Switzerland, Germany) assume a lower occupant load density for base/upper levels. Other directives (e.g. New Zealand, UK) assume the opposite. And for others (Japan, Canada, IBC) there is no distinction at all. However, no justification for these differing interpretations could be found in the literature. It is also reasonable to conclude that

1 Railway stations, airports and other public transport terminals that offer goods and services primarily oriented to the needs of travellers. The term "public transport terminals" covers large starting and ending stations with a large number of passengers. The definition of a large number of passengers is specified for cantonal or urban transport hubs. For example, the city of Zurich sets a value of at least 15'000 passengers per day for public transport terminals.

2 In Switzerland, Sunday sales are only permitted with an exemption permission usually at large public transport terminals.

3 Not to be mistaken for the occupant load density. The data for determining this value can be found in Figure 12.

the definition of different occupant load densities implicitly imposes stricter or laxer requirements on the design of the means of egress.

According to the authors, the origin of these values cannot be accurately traced. Certain densities can be tracked back to an extensive survey by Courtney et al. [2] from 1935 (!). In this study the occupant load densities for the retail stores were estimated based on an interview with a sales manager, in which he or her was asked to approximate the maximal occupant load density for his or her retail store. The densities estimated by Courtney et al. can still be found in various guidelines such as NFPA 101 [3] and IBC [4].

However, it should be noted that most of these occupant load densities are intended to be used within a prescriptive regulation for design of the means of egress. It is possible that, depending on the design format of the prescriptive code, smaller densities may nevertheless lead to wider exits – and therefore to a higher level of safety. This could apply, for instance, when regulations contain a larger codified required exit width per occupant. It is therefore difficult to make a comprehensive international comparison on the level of safety when assessing only the occupant load density.

Performance-based design approaches, on the other hand, aim to generate a more realistic picture of the fire risk in buildings. For performance-based design approaches, the design values for occupant load densities are often borrowed from prescriptive regulations. Nevertheless, these values usually do not represent a realistic occupant load scenario. Providing design values for performance-based design approaches that are related to realistic conditions is therefore very important when applying a performance-based design approach.

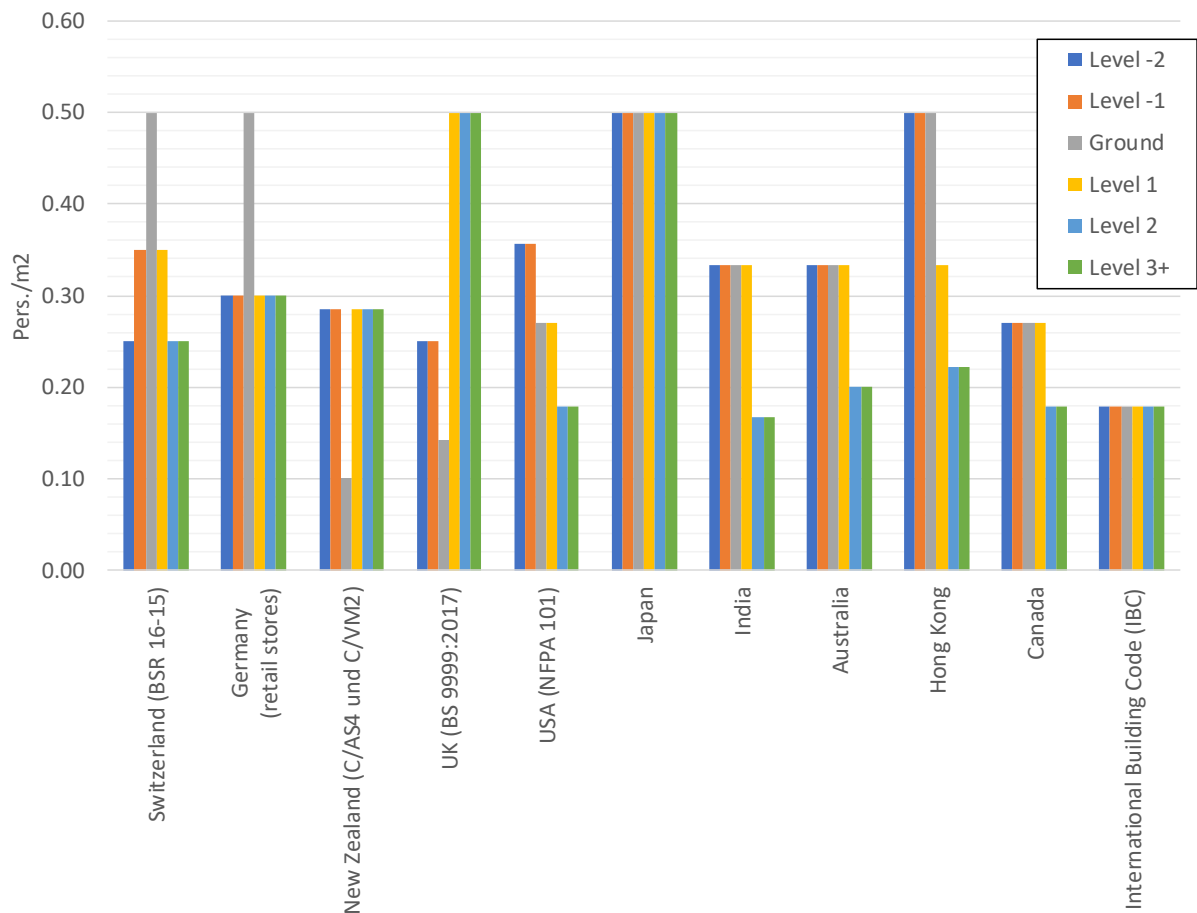


Figure 2: Compilation of the normative occupant load densities in different countries (according to information according to [1,5,6]).

2.3 Review of surveys on the occupant load densities in retail stores

Regardless of a building's use, the occupant load is one of the most important variables for assessing life safety. For this reason, numerous surveys have been carried out in the past for various building types. Most of these studies are based on counting actual occupants during inspections, e.g. counting the current number of building users or the number of workplaces. A similar methodology can be applied in buildings where the maximum number of occupants is strongly influenced by the capacity of the building, e.g. residential buildings [7], hotels, hospitals, school buildings, theatres, cinemas, etc. Courtney et al. [2] conducted surveys for most of the occupancy types listed as early as 1935.

However, the number of people in retail stores is strongly influenced by each customer's individual choice and desire to visit the store. This leads to a high temporal variability of occupant density. For this reason, random inspections are not useful for determining the density of people in sales outlets. Angerd [8] evaluated the occupant load density during peak sales days of the year. This derived density can then be used as a design value for performance-based design approaches. However, the disadvantage of this approach is that the total variability of the number of persons remains unknown and makes it unsuitable for risk-based approaches.

Charters et al. [9] analysed the estimated occupant load density reported by the fire brigade in retail stores in the event of fires in Great Britain. These values correspond to the observed occupant load density when the evacuation request by the fire brigade took place. However, it is unclear how reliable the data is and whether the fire brigade provided only rough estimates

or an actual count of persons present. For example, one fire event reported an occupant load density of 0.5 pers./m². It is unclear what circumstances of the fire event lead to such a high value. However, it corresponds to the British standard value at that time, making it possible that the value was used merely for data collection purposes instead of representing an actual counting of occupants. However, Charters et al. also recognised a large distribution in occupant load density. Apart from the previously mentioned value, the other 15 investigated fire cases reported much lower values of 0.1 pers./m² (approx. 75%) and 0.2 pers./m² (approx. 19%).

De Sanctis et al. [10] analysed occupant load densities in Swiss retail stores based on various data sources, including receipts (sales made with receipts) and daily measurements by various counting systems. However, the estimation of the occupant load density had to be made whilst taking into account large uncertainties, e.g. the length of stay of customers, the number of persons per receipt, and the temporal distribution of purchases. The aim was to determine the variability of occupant load density and to demonstrate the need for more accurate data collection to help determine the occupant load density in retail stores. It was postulated that a considerable reduction of the occupant load density for retail stores of the Swiss fire safety regulation could be achieved and laid to the foundation for the present survey.

2.4 Fire events in retail stores

The number of people present in the event of fire is particularly important for life safety considerations. Since fires can be directly caused by human action, the occupancy load and ignition of the fire might not be independent. For this reason, it is important to analyse a potential dependency between occupancy and frequency of fires in order to determine the occupant load density in retail stores.

For several years, the Intercantonal Reinsurance Association (IRV) in Switzerland has maintained a database with fire incidents in buildings and their consequences (personal injury and damage to buildings). Several different static evaluations of fire causes have been performed using this database [11,12]. The statistical evaluation of the fires was based on the building use defined by the IRV and the cause of the fire, with retail stores falling under the building use "Trading". In this category a distinction was made between fires in retail store, fires in commercial and department stores, and fires in warehouses.

Fire deaths and fire causes in retail stores

Fischer et al. [12] analysed the loss statistics of the IRV for the period between 2000 and 2007 with regard to the overall number of fatalities in fires (Figure 3). In this period the fatality rate was found to be less than 1% of all observed fatalities over all building uses. Therefore, deaths due to fires in retail stores are very rare in comparison to fire deaths in other building uses (especially in residential buildings).

The cause of fire for the building use "Trading" was also investigated by Fischer et al. [12] (see Figure 4), with arson being identified as the main cause. Since arson comprises a human factor it may be related to the occupant load. However, the building use category "Trading" also involves fires in commercial buildings as well as warehouses. The percentage of fires in retail stores within this category is unclear.

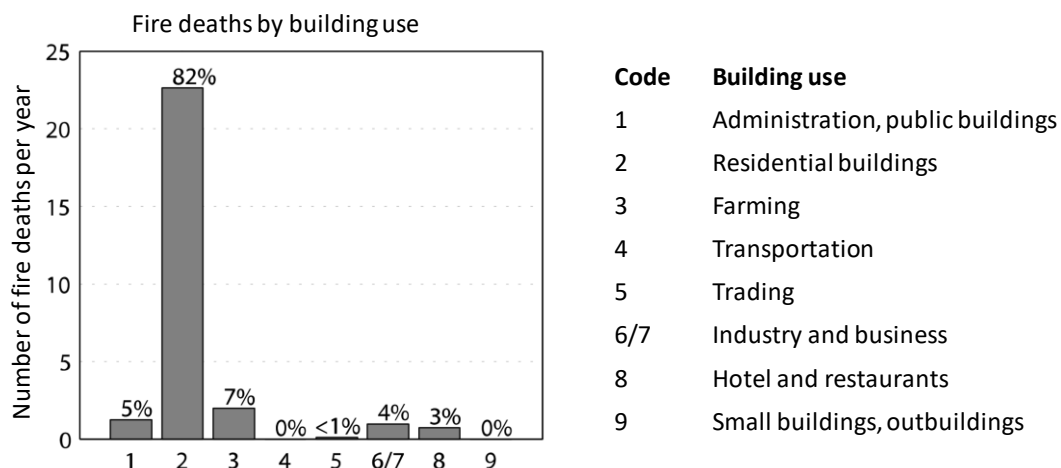


Figure 3: Number of civilian fire deaths per year by building use (Period 2000-2007, 8-year average, from [12]).

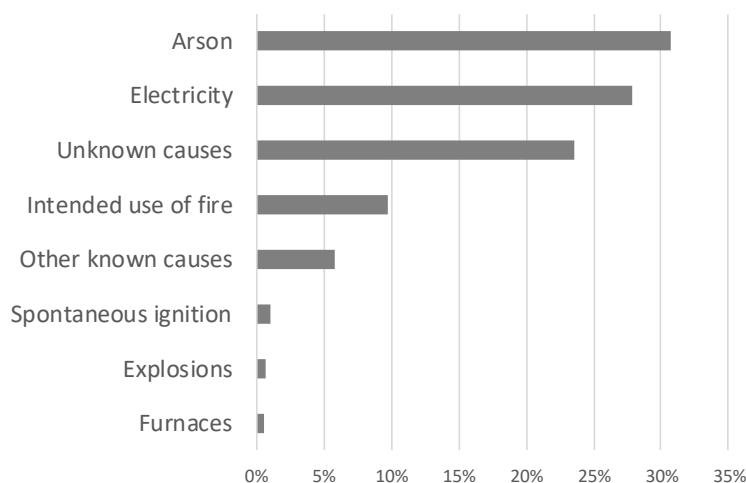


Figure 4: Causes of fires for building use "Trading" (from [12], excluding lightning strikes).

A more detailed survey on the causes of fire in retail stores is provided by one of the two major retailers in Switzerland. Fire events have been recorded by them since 2010, with a case description recorded for each event. During the period from 2010 to 2016, 35 fires were registered (excluding lightning strikes), though only fires in publicly accessible areas (e.g. sales outlets, DIY stores, restaurants and cafés, but not pick-up areas and non-public car parks) were considered. Information provided about these fires include:

- Cause of fire (same classification as the IRV database)
- Date of the fire
- Associated selling area
- Type of retail store (Supermarket, DIY, gastronomy, etc.)
- Description of the event

During this period, no deaths were recorded in retail stores, which supports the statement that deaths due to fires in retail stores are very unlikely in Switzerland. Figure 5 illustrates the frequency of each cause of fire for the events recorded. For retail stores arson is the third most common cause. Except arson and (probably) the unknown causes, the remaining causes (electricity, spontaneous ignition, and intended use of fire) have less to do with the occupancy load

in retail building. Indeed, a dependency between occupant load and fire can only be expected in a maximum of 60% of all fires.

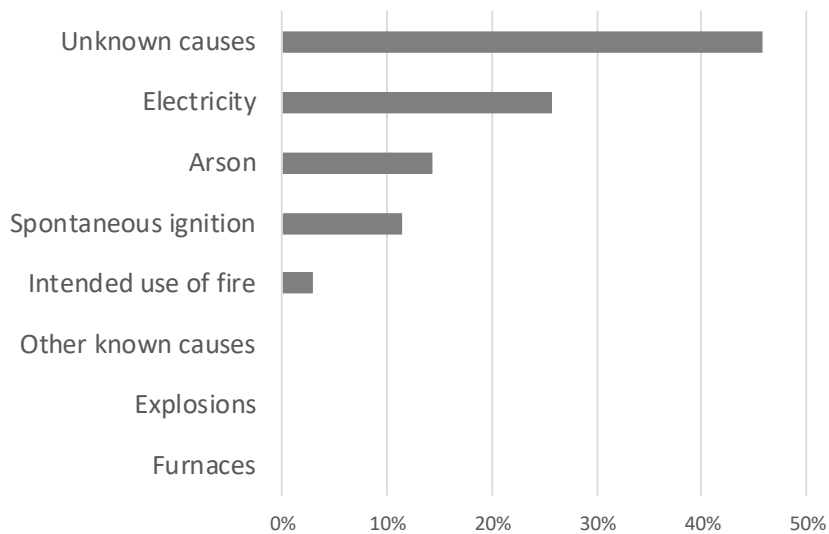


Figure 5: Causes of fires for retail stores recorded by one of the two major retailers in Switzerland (2010-2016).

Dependency between occupant load and fire ignition

Since people can cause fires through carelessness or even the normal use of a building's facilities, there could be a dependency between a high number of people (as potential initiator of a fire) and fire frequency. The literature has paid only limited attention to this dependency, as the worst-case scenario is usually considered, e.g. a high occupancy load simultaneous with a large fire. From a statistical point of view, fires can occur at any time and can be described by the fire ignition frequency. There could be a dependency between the number of people in a store and the frequency of a fire [13]: If the number of occupants changes, the probability of a fire is also affected. As far as the authors are aware, no publication has yet addressed this issue for retail stores. Bennetts and Thomas [13] discussed the effect of the number of people on the fire ignition frequency in office buildings. They concluded that the fire frequency increases during operating hours, i.e. during the use of the building, without relating it to the number of occupants. However, they found that at the same time the likelihood of a fire remaining small increased, as people could detect and fight the fire more quickly. A higher number of people can therefore have both positive and negative effects.

In order to investigate the dependency between the number of people and the number of fires, the fire data from a major retailer in Switzerland is evaluated by month and by day of the week. The evaluation differentiating by month (Figure 6) shows that most fires occurred in November. When taking a closer look at the causes of the fires in November, however, one finds no evidence to suggest that the fires could be associated with an increased number of people (mainly unknown causes and electric fires).

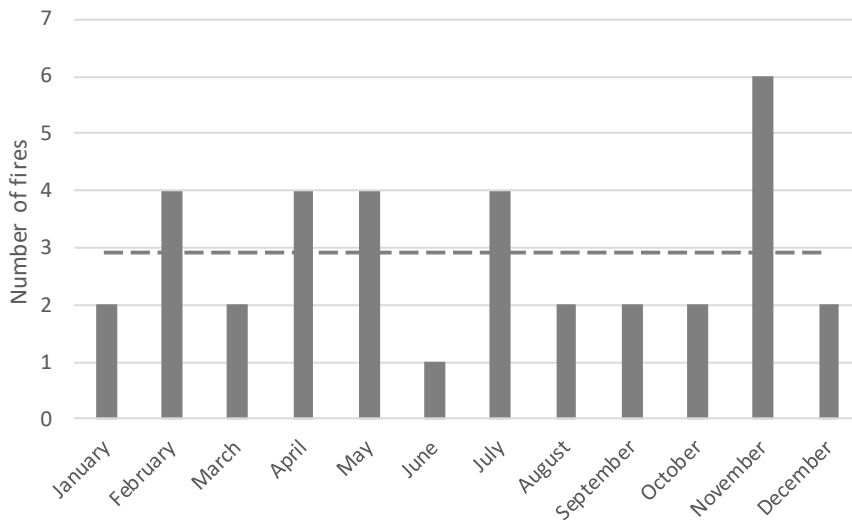


Figure 6: Fire events per months.

The evaluation by weekday (Figure 7) shows that Mondays and Saturdays have the most recorded fire events, though this higher frequency is not statistically significant. However, especially on Saturdays, an increased number of people can be expected in Switzerland. The analysis of the causes of fires on Saturdays provide no indication of the existence of a dependency between the number of persons and the number of fires. Among all occurred fires the causes of fires on Saturdays were:

- 2 electric fires (defective refrigerator / cleaning machine)
- 2 arson
- 2 unknown causes (fire in bicycle shelter / smoke development in retail store)
- 1 intended fire (fire on a terrace of a rented apartment as part of the retail store)

It should be noted that the frequency of arson is higher on the weekends; four out of five arson cases took place between Friday and Sunday. These fires are mainly due to vandalism and are more likely to occur in the evening after closing time of the store. Electric fires occurred mainly due to defective electrical appliances and can be regarded as independent of the occupant load.

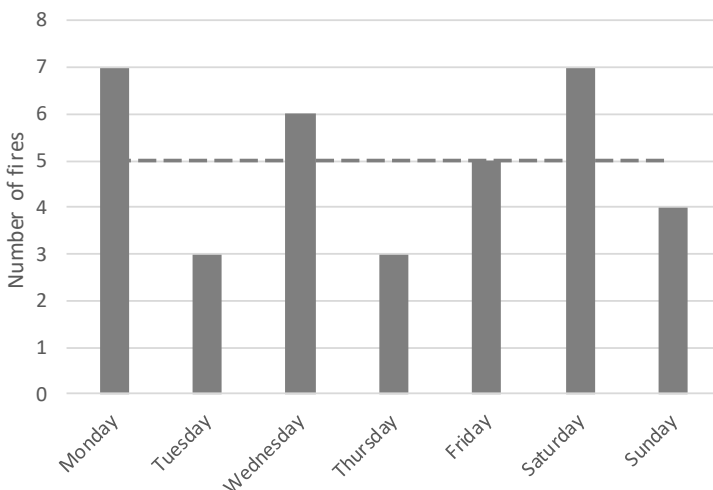


Figure 7: Fire events per weekday.

To summarise, it cannot be conclusively assessed whether there is a correlation between the number of persons and the fire ignition frequency. They are probably weakly correlated or even independent. A strong correlation can practically be excluded due to the types of causes and the temporal distribution of the fires. A strong correlation would indicate a need for a survey assessing annual or daily maximum values of the occupant load. Because of the weak correlation, it is justifiable to record the occupant load density in a high temporal resolution (e.g. as point-in-time), e.g. in minute-by-minute steps, when the data is applied for fire safety problems.

3. Survey of the occupant load density in retail stores

This chapter explains how the occupant load density in retail stores can be assessed in general. First, methods to assess the occupant load density by observational measurements are presented. In the second part of the chapter, different measurement systems are reviewed, with the third part discussing how to deal with measurement errors.

3.1 General assessment method of the occupant load density

The occupant load density d can be defined using the ratio of the number of persons p that are present in a compartment and its net floor area a_f (Eq. 1). Here, the occupant load density is measured in persons per square meter [pers./m²].

$$d = \frac{p}{a_f} \quad (1)$$

The number of persons in a room p [pers.] varies in time. Hence, p can be represented as a random process $P(t)$. The reference period for this random process is set to one year, assuming stationarity and ergodicity over this period.

Usually, the number of persons in a room cannot be counted directly and must be derived by other measurements. Two methods can be applied:

- Method A: The occupant load is derived based on the counting of arrivals and departures.
- Method B: The occupant load is derived based on the counting of arrivals (or departures) and the measurements or information available regarding the length of time customers spend at the store (dwell time).

Method A: Assessment based on the number of arrivals and departures

The random process for the number of persons $P(t)$ in a compartment can be described by two underlying processes, one for the arrivals and the other for the departures of persons [10]. The first process is represented by the cumulative number of arrivals $N_A(t)$ and the second process by the cumulative number of departures of the persons $N_D(t)$ over the span of one day. Two realisations of the processes $n_A(t)$ and $n_D(t)$ are illustrated in Figure 8. The number of persons present in a store $P(t)$ at any time t can be assessed by:

$$P(t) = N_A(t) - N_D(t) \quad (2)$$

In order to use this method, systems must be used that are able to distinguish between arriving and departing persons. Alternatively, the systems can also be applied at locations where the flow of people is unidirectional.

Method B: Assessment based on the dwell time

A realisation of the cumulative number of arrivals $n_A(t)$ depends on the arrival time $t_{A,i}$ of the i th customer. This time can be considered as random and is represented through a random variable $T_{A,i}$. For each realisation of this time $t_{A,i}$ the cumulative number of arrivals $n_A(t_{A,i})$ increases by one. The time a customer stays in the room is denoted as the dwell time τ_i and represents the time from the arrival of a customer until the time of departure of the same customer. This time is individual and varies from customer to customer. For each customer i the departure time $t_{D,i}$ is assessed through:

$$t_{D,i} = t_{A,i} + \tau_i \quad (3)$$

By reordering the departure times $t_{D,i}$ in ascending order the cumulative number of departures $n_D(t)$ is increased by one at each departure time $t_{D,i}$. In this way the number of departures never

exceeds the number of arrivals ($n_A(t) \geq n_D(t)$). The occupant load is derived based on the measured cumulative number of arrivals $N_A(t_{A,i})$ and the calculated cumulative number of departures $N_D(t_{D,i})$ by Eq. 2. The method can also be applied vice versa by measuring the number of departures and calculate the number of arriving persons over the dwell time.

In order to use this method, systems can be used that a) count the persons arriving or leaving and b) measure dwell time for each occupant, e.g. by tracking systems. The advantage of this method is that information on entries or exits is generally available (e.g. counting by light barriers, or by timestamp of purchase receipts), so that the occupant load density can simply be estimated with assumptions regarding the dwell time.

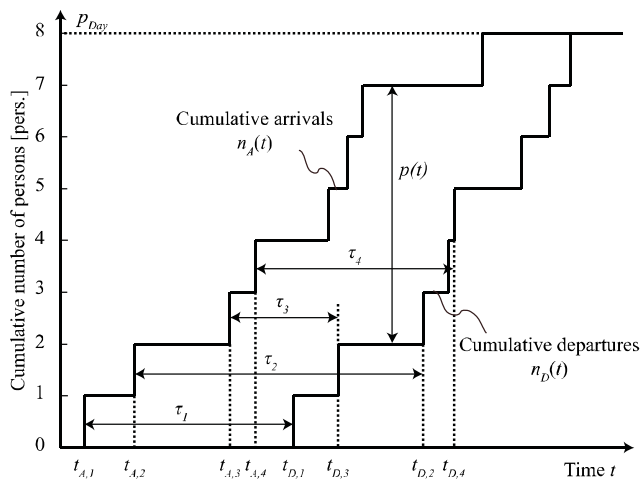


Figure 8: Cumulative arrivals and departure to and from a system for 8 customers as an example [10].

3.2 Measurement systems

In Table 1, sensor systems for the counting of persons are listed and compared. For the application to counting visitors in retail stores, optical sensors are judged to be best suited due to the high accuracy and cost-value ratio.

Technology	Application	Advantages	Disadvantages
optical sensor	for indoor and outdoor use at different heights; can be used for tracking and barrier measurement	very accurate even at high occupancy load densities; flexible mounting height due to different lenses	not working with visual obstacles
WiFi and bluetooth sensor	for indoor and outdoor tracking	very high range; low cost	very inaccurate
thermal sensor	for outdoor use; suitable for poor visibility conditions such as fog, smoke or darkness	no ambient light necessary	high cost, no optical zoom possible; requires heating at low temperatures
ultrasonic sensor	for indoor and outdoor use; as barrier measurement at entrances	low cost	very susceptible to external interference; highly inaccurate
radar sensor	for indoor and outdoor use; as barrier measurement at entrances	low power consumption	tracking not possible; overlapping objects are not detected; can be disturbed by other electromagnetic waves
infrared sensor	for indoor and outdoor use; as barrier measurement	very low cost	tracking not possible; overlapping objects are not detected; very short range
laser sensor	for indoor and outdoor use; as barrier measurement	very high range	requires heating at low temperatures; high cost

Table 1: Comparison of sensor technologies for person counting.

3.3 Measurement errors

Balancing entries and exits

Measurement errors can lead to flawed results when assessing the occupant load, e.g. by falsely counting too many entrances or too few exits. However, as a basic rule, the number of entries must be the same as the number of exits from the beginning to the end of the store's opening hours. The measurement error can be reduced, if entries and exits of persons at all entrances are balanced. It must be noted that the measurement error increases over the course of the day. In order to optimize the measurement accuracy, the balance is therefore extrapolated forwards and backwards in time. This results in two graphs which are combined with each other:

$$L = w_{FW} \cdot L_{FW} + w_{BW} \cdot L_{BW}$$

$$L_{FW_t} = L_{FW_{t-1}} + IN_t - OUT_t$$

$$L_{BW_t} = L_{BW_{t+1}} + OUT_{t+1} - IN_{t+1}$$

$$w_{BW_t} = \frac{\sum_{i=0}^t IN_i + OUT_t}{\sum_{i=0}^t IN_i + OUT_i}$$

$$w_{FW} = 1 - w_{BW}$$

Measurement error

The difference between the number of entrants and the number of exits is used for the error estimation. The relative measurement error of the balancing is therefore as follows:

$$\delta = \frac{|IN - OUT|}{IN + OUT}$$

However, the absolute error per square metre is relevant for the occupant load density. The absolute error increases linearly with the number of visitors per square metre. By calculating occupant load forwards and backwards in time, the maximum error is reached in the middle instead of at the end of the day, thus halving the measurement error (Figure 9). The error estimation for the occupant load density is:

$$\Delta = \frac{|IN - OUT|}{2 \cdot A}$$

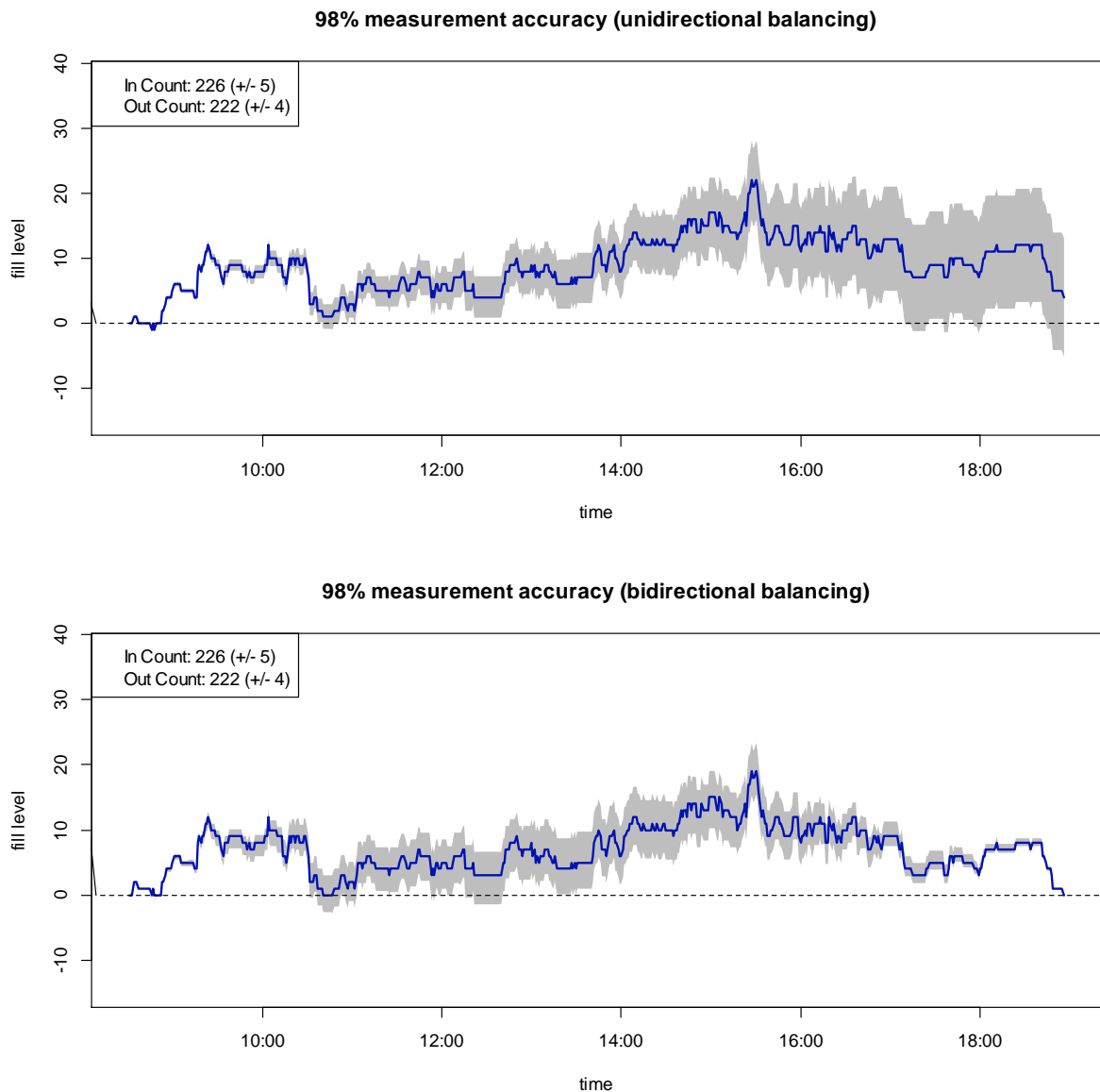


Figure 9 Visualisation of error correction for unidirectional balancing (above) and for bidirectional balancing (below).

4. Occupant load density in Swiss retail stores

The aim of the survey is to provide a comprehensive figure of the occupant load density in Swiss retail stores. The survey includes different types of retail stores that are normally to highly frequented. According to Chapter 2.4, the occupancy load and the fire ignition frequency tend to be uncorrelated or possibly even independent. Therefore, it is very likely that a fire does not occur at peak times but at a random time with a random occupant load. In order to provide a satisfactory statistical distribution of the occupant load density, the time resolution of the measurement should be high. This statistical distribution can be used as a basis for the determination of design values for performance-based design approaches but also for the verification of prescriptive design requirements for the means of egress.

4.1 Representative sample

A list of the stores involved in the survey and their locations can be found in Appendix A1. For reasons of confidentiality, however, identifying information has been excluded. Care was taken to ensure that all characteristics of the stores that could have an influence on the occupant load density were included, e.g.:

- type or category of use of the store
- size of sales areas
- location (branches in rural areas, in the agglomeration, and in urban areas)
- user frequency (highly frequented and normally frequented retail stores)
- floor level

4.1.1 Retail types

A total of 96 stores from 6 different organisations (Coop, Lib AG, Maus Frères, Migros, Möbel Pfister and Volg) were examined over a measurement period of a year. The retail stores were divided into 13 different retail types. Figure 10 shows the distribution of retail types for one of the two major retailers in Switzerland. Almost 75% of the retail stores are supermarkets, i.e. sales outlets with a strong focus on food sales. Other non-food stores account for about 25% of the stores. Therefore, it makes sense to take a closer look at supermarkets. The supermarkets are divided into 5 different categories based on their sales areas (see also Chapter 4.1.2). The following abbreviations are used for the sake of clarity:

sales areas up to	600 m ² :	Supermarket_600
sales areas from	601 m ² to 1200 m ² :	Supermarket_1200
sales areas from	1201 m ² to 2400 m ² :	Supermarket_2400
sales areas from	2401 m ² to 4800 m ² :	Supermarket_4800
sales areas larger	than 4800 m ² :	Supermarket_large

Figure 11 shows the resulting number of recorded stores per retail type.

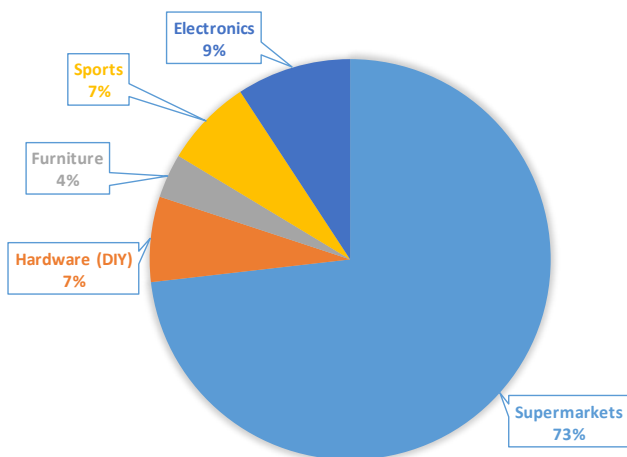


Figure 10: Distribution of retail types of the basic population of retail stores for one of the two major retailers in Switzerland for the year 2016 by the number of stores.

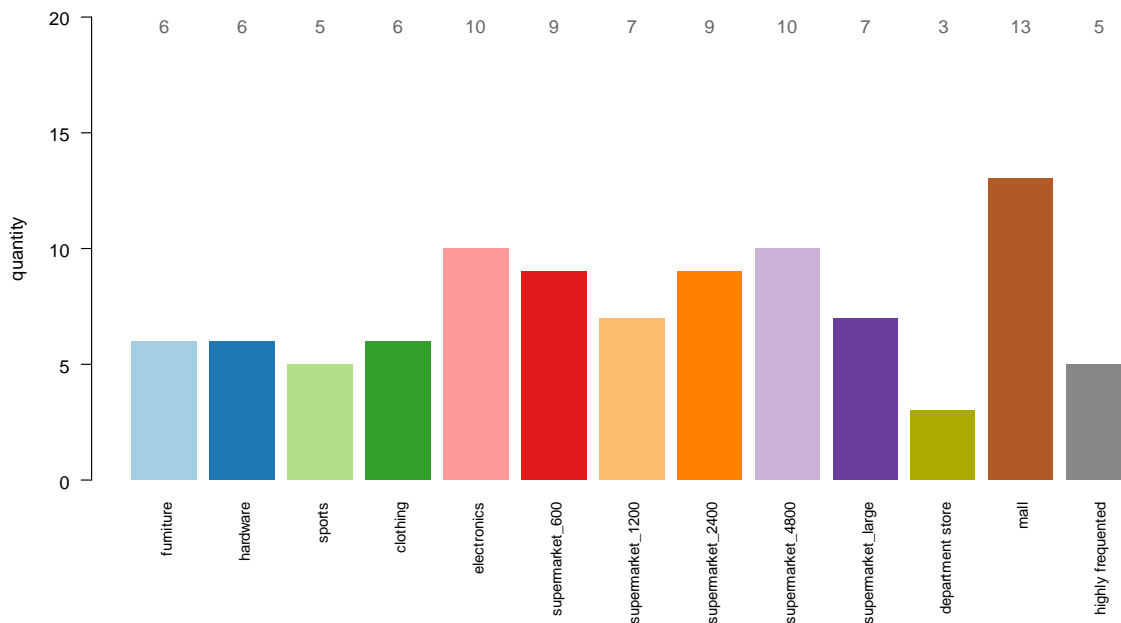


Figure 11: Number of stores per retail type.

Category highly frequented supermarkets

Among the 47 supermarkets, there are 5 stores located at important public traffic hubs with a Sunday sale, which in Switzerland are only permitted in special cases. These stores are associated with a very high visitor frequency per store area. A few high-frequency stores were explicitly selected in order to investigate extreme occupancy rates. For this reason, a separate "high-frequency" category is used for these highly frequented stores.

Figure 12 shows the average total visitor frequency per day and square metre (not to be confused with the occupant load density). The category "high frequency" is clearly different from the others. No branch in the high frequency category has an average total of less than 7 visitors per m² of sales area per day. At the same time, this value is not reached by any other category. This value can be taken as a quantitative criterion for distinguishing between normally frequented supermarkets and highly frequented supermarkets. For new supermarkets, however,

the daily visitor frequency could be estimated by a market analysis to decide whether it fell into the category “highly frequented supermarkets”.

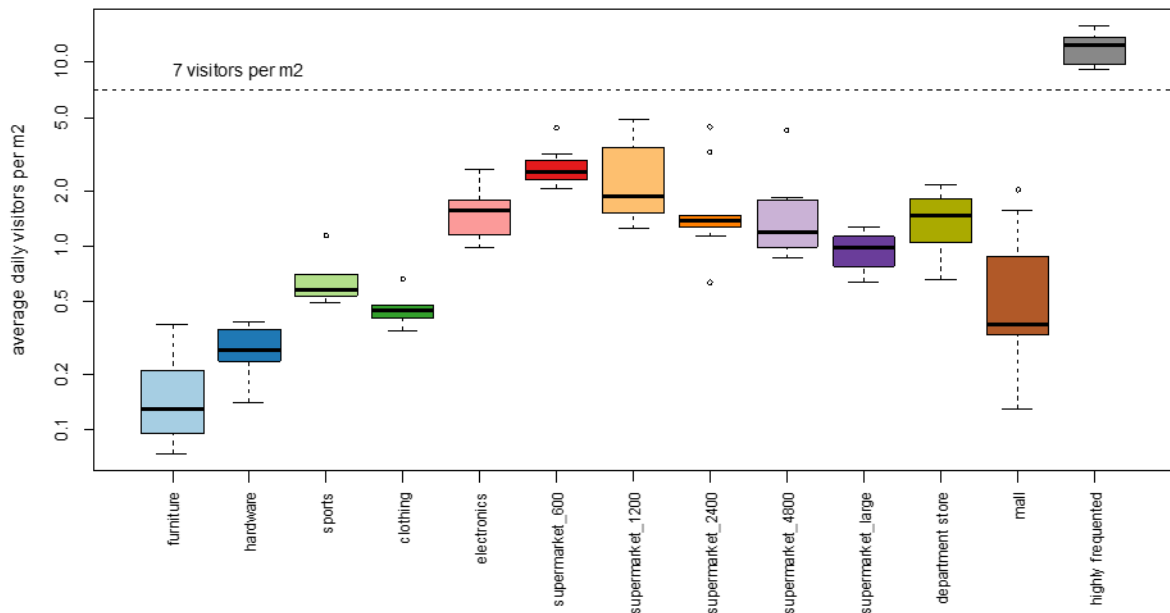


Figure 12: Average total visitor frequency per day and square metre.

4.1.2 Sales area

The sales area is relevant for the determination of the occupant load density. To investigate whether the occupant load density is influenced by the sales area, the category supermarket is subdivided in different subcategories with different sales areas. Figure 13 shows the distribution of sales areas per retail type. Since the supermarkets are divided into space categories, the sales areas spread within the limits of the category. A subdivision for the other retail types was omitted. This means that in some cases the distribution of the sales area is somewhat larger compared with the supermarkets.

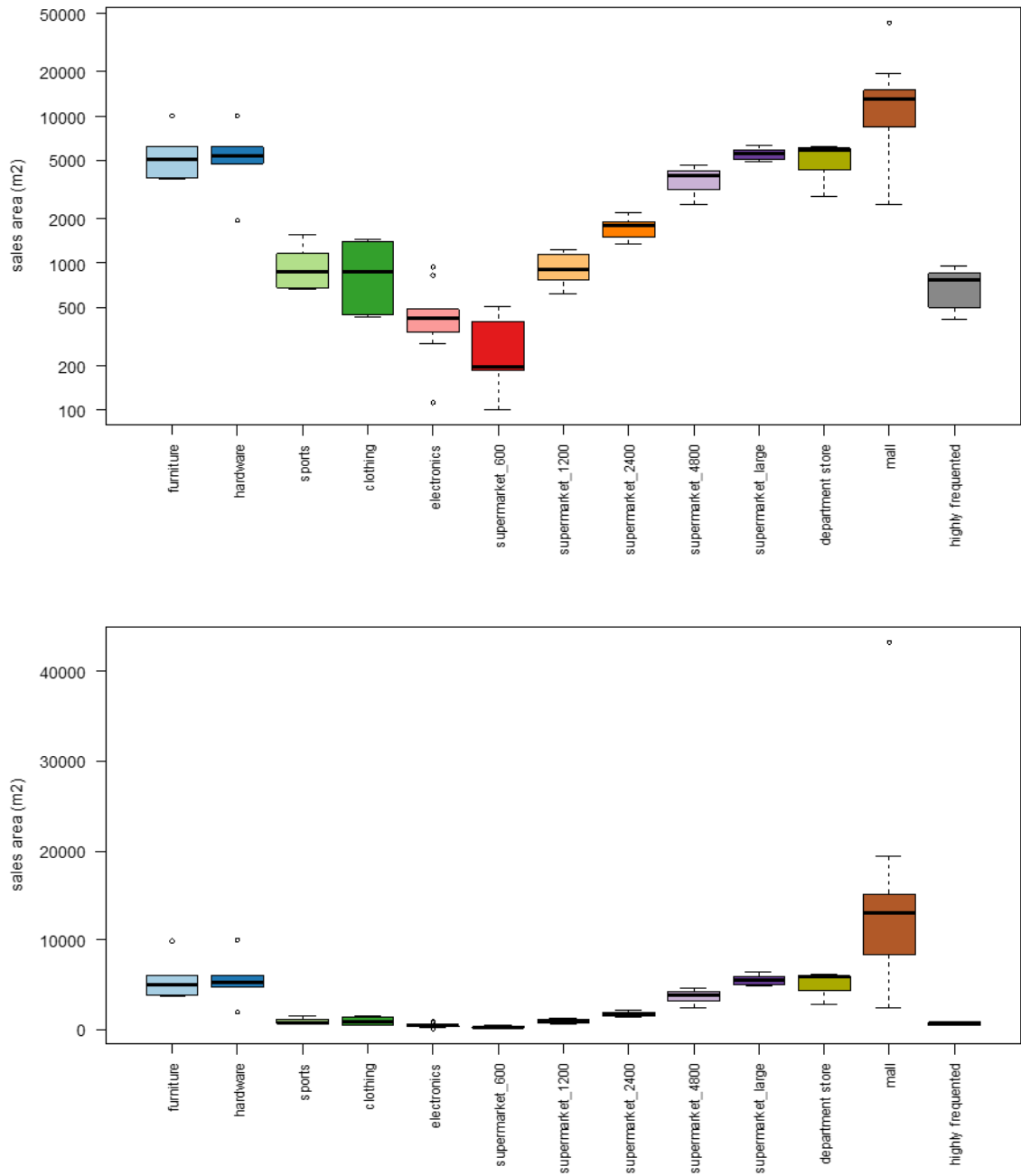


Figure 13. Distribution of sales areas per retail type by logarithmic scale (top) and by normal scale (bottom).

Supermarkets

Figure 14 shows the distribution of the supermarket sales areas of one of the two major retailers in Switzerland. Nearly 80% of the surveyed stores have sales areas between 600 m² and 2,400 m². This can be explained on the one hand by increased prescriptive fire protection requirements (e.g. need for a sprinkler above 2,400 m² and/or need for an electro-acoustic emergency system for multi-storey shops). On the other hand, there is limited space in cities and therefore there are lots of smaller supermarkets. Hence, it makes sense to define space-related subcategories for supermarkets.

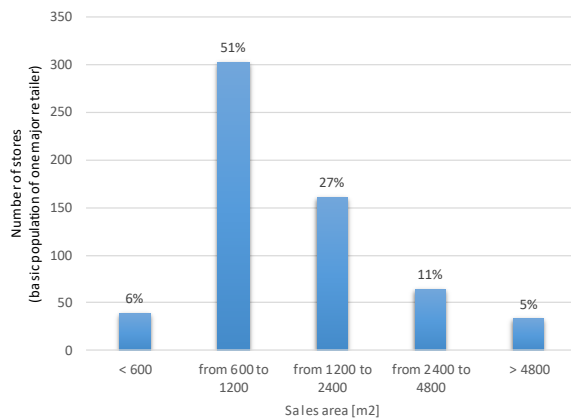


Figure 14: Distribution of sales area for supermarkets for one of the two major retailers in Switzerland.

Department stores and malls

Malls are characterised by a mixed use of different retail types and by large traffic areas. The latter can vary greatly between individual malls and are often also used as promotion areas. Therefore, the measurements in malls took place over a predefined area, which also includes circulation areas. The occupant load density is assessed by dividing the occupant load by the gross sales area within the perimeter and does not include the circulation areas. Thus, the case considered is one in which all visitors are standing within the sales areas. In this way the occupant load density in malls is overestimated, since the occupant load is divided by a smaller area. The occupant load density resulting from special events in malls (promotion, concerts, etc.) is not considered, since the perimeter is defined in such a way that it does not cover these event areas.

For department stores, the measurement took place either throughout the whole building or per floor level.

4.1.3 Representation of the basic population

The stores for the survey were selected based on measurement systems that had already been installed and based on an assessment of how representative the store is for the determination of the occupant load density. In order to verify quantitatively whether the sample represents the basic population, the area-related revenues of sample stores were compared with the area-related revenues of stores from the basic population. It is assumed that the number of visitors correlates strongly with revenues in supermarkets (the more people, the more revenue). The comparison is illustrated in Figure 15 and shows that, on average, the sample fits the area-related revenues of the basic population very well. Thus, it can be assumed that the sample represents the basic population to a sufficient degree.

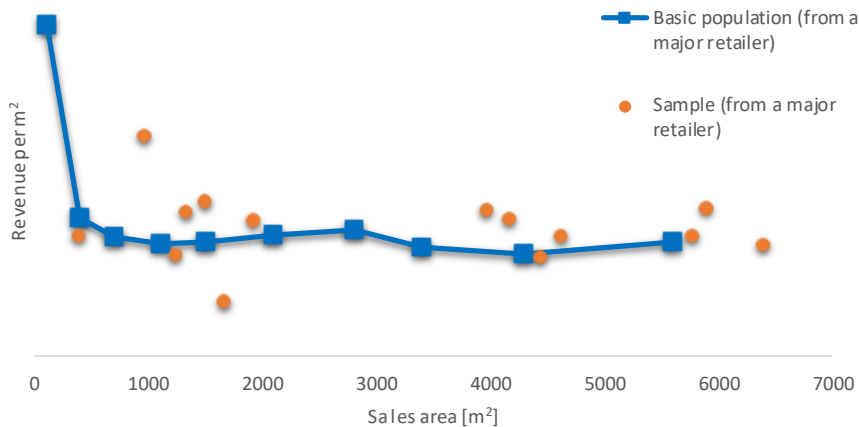


Figure 15: Area-related revenue of the basic sample and of the supermarkets of one of the two major retailers in Switzerland.⁴

4.2 Measurement concept and requirements

The occupant load density is determined in minute-by-minute steps by measuring the flow of persons at the entrances and exits of the sales areas (Method A, see Chapter 3.1).

The measurements were commissioned by the survey participants, e.g. the store managers. Since different measurement systems from different manufacturers were liable to be used in the survey, measurement requirements were formulated to ensure data quality. Annex A2 describes the requirements for the measurement concept, the documentation, and the defined criteria.

4.3 Statistical evaluation of the occupant load density

Two different approaches are used for the evaluation:

Method 1: Determination of quantile-values per store and its statistical evaluation for each retail type.

Method 2: Grouping of the evaluated minutes per retail type (over all stores) and its statistical evaluation.

In the first method, an evaluation of the occupant load density is carried out for individual stores. This evaluation method is suitable when disregarding the whole basic population.

However, since the sample represent the basic population, in the second procedure, this individual evaluation does not take place. Instead, the evaluation is carried out for all stores for the whole length of operating time. As a result, the overall representative statistical distribution of the occupant load density can be derived per retail type and allows for an application to stores outside of the sample.

4.3.1 Evaluation of quantile values per store (Method 1)

A 99% quantile value is considered for the evaluation, e.g. the value of the occupant load density that is exceeded in 1% of cases. Figure 16 shows the distribution of 99% quantiles per store for all retail types. The number of stores per retail type and the 99% quantile of occupant

⁴ For confidentiality reasons, a presentation of the revenue figures is omitted and only the qualitative course is shown.

load density is documented in Table 2. The analysis shows that supermarkets (especially the highly frequented supermarkets) achieve the highest densities. Of these, only one highly frequented supermarket reaches a 99% quantile of more than 0.35 pers./m². This is followed by some supermarkets with a 99% quantile of up to 0.25 pers/m². The 99% quantiles of most of the remaining stores are smaller than 0.1 pers./m².

Retail type	99% values of the occupant load density [pers./m ²]									
	0 - 0.05	0.05 - 0.1	0.1 - 0.15	0.15 - 0.2	0.2 - 0.25	0.25 - 0.3	0.3 - 0.35	0.35 - 0.4	0.4 - 0.45	0.45 - 0.5
Furnitures	6	0	0	0	0	0	0	0	0	0
Hardware (DIY)	6	0	0	0	0	0	0	0	0	0
Sports	5	0	0	0	0	0	0	0	0	0
Clothing	6	0	0	0	0	0	0	0	0	0
Electronics	2	8	0	0	0	0	0	0	0	0
Supermarket_600	0	7	2	0	0	0	0	0	0	0
Supermarket_1200	0	6	1	0	0	0	0	0	0	0
Supermarket_2400	1	7	0	0	1	0	0	0	0	0
Supermarket_4800	0	10	0	0	0	0	0	0	0	0
Supermarket_large	3	3	1	0	0	0	0	0	0	0
Department store	1	1	1	0	0	0	0	0	0	0
Mall	9	2	2	0	0	0	0	0	0	0
Highly frequented	0	0	0	1	0	3	0	1	0	0

Table 2: Number of stores per retail type and 99% quantile value of the occupant load density.

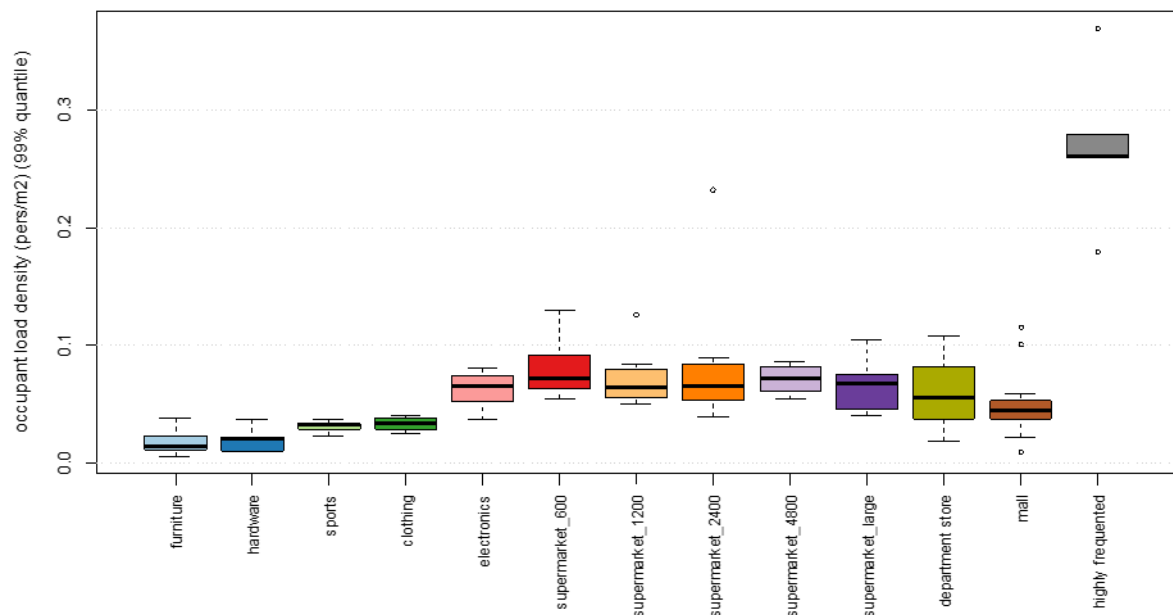


Figure 16: Box-Plot of the 99% quantile per retail type (Method 1).

4.3.2 Evaluation per retail type (Method 2)

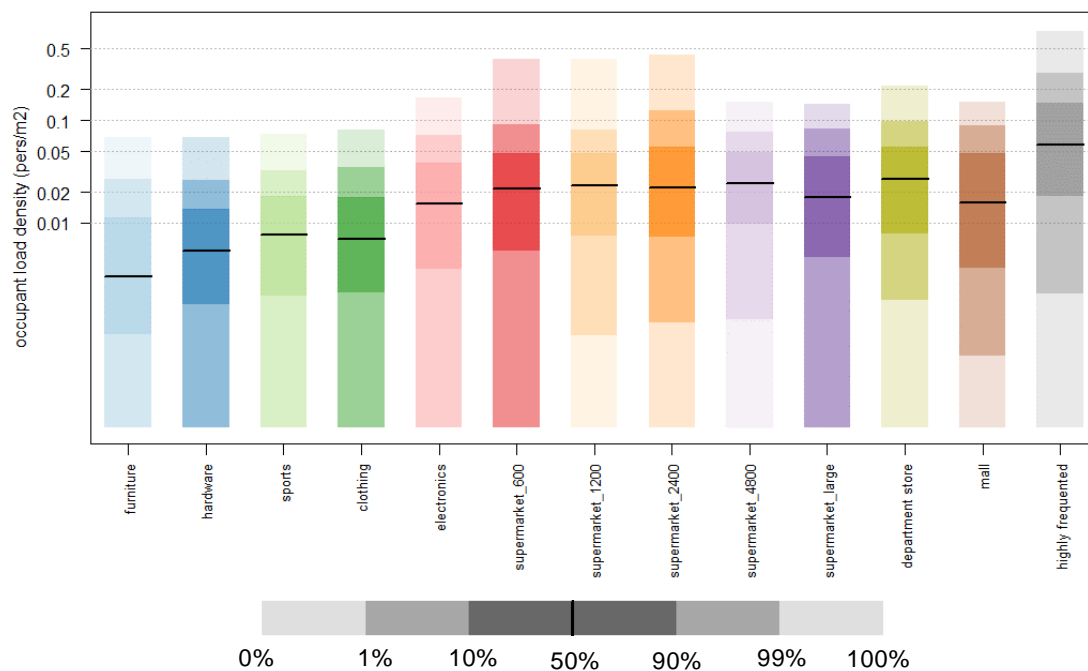
The quantiles of the occupant load density per retail type are shown in Table 3. Figure 17 shows various quantiles of the occupant load density per user category. The 99% quantile is below 0.3 persons/m² over all categories. Only the highly frequented supermarkets exceed the occupant load density of 0.15 pers./m² in the 99% quantile.

Figure 18 to Figure 20 show the inverted empirical cumulative probability distribution of the occupant load density. The 99% quantile is indicated as a dashed line. There are very large differences between the categories. While in the highly frequented supermarkets a maximal

density of up to 0.74 pers./m² is reached, the maximum values of the remaining supermarkets are below 0.45 pers./m². Furthermore, the maximum values of the specialist stores are all below 0.2 pers./m². The 99% quantile value of the whole sample is mainly influenced by only one to a few stores, since only 1% of the measured densities of the basic population are relevant. This also results in the distribution function of the Supermarket_600 category being stepped shape: the data for the occupant load density above the 99% quantile comes from a single store with a small sales area of 100 m², which leads to the coarse resolution in the density.

Retail type	Quantile						
	0%	1%	10%	50%	90%	99%	100%
Furnitures	0.000	0.000	0.001	0.003	0.011	0.027	0.068
Hardware (DIY)	0.000	0.000	0.002	0.005	0.013	0.026	0.068
Sports	0.000	0.000	0.002	0.008	0.018	0.033	0.073
Clothings	0.000	0.000	0.002	0.007	0.018	0.035	0.081
Electronics	0.000	0.000	0.004	0.015	0.039	0.071	0.165
Supermarket_600	0.000	0.000	0.005	0.022	0.048	0.091	0.400
Supermarket_1200	0.000	0.001	0.007	0.023	0.048	0.080	0.396
Supermarket_2400	0.000	0.001	0.007	0.022	0.055	0.126	0.438
Supermarket_4800	0.000	0.001	0.010	0.025	0.048	0.077	0.151
Supermarket_large	0.000	0.000	0.005	0.018	0.044	0.082	0.145
Department store	0.000	0.002	0.008	0.027	0.055	0.099	0.218
Mall	0.000	0.001	0.004	0.016	0.047	0.088	0.150
Highly frequented	0.000	0.002	0.018	0.057	0.147	0.286	0.743

Table 3: Quantiles of the occupant load density per retail type.



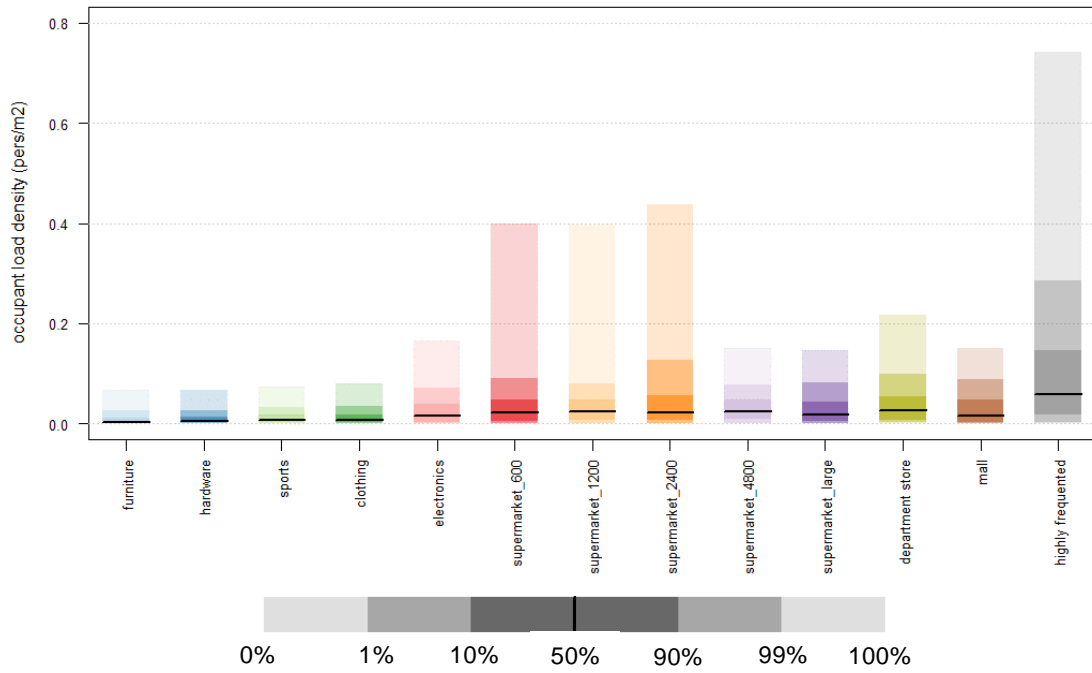


Figure 17: Quantiles of the occupant load density per retail type by a logarithmic scale (top) and by a normal scale (bottom)

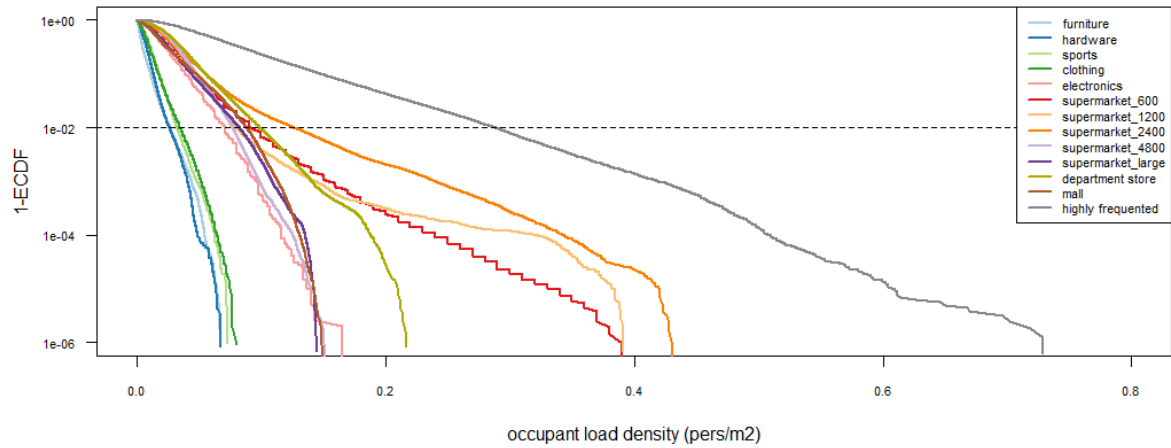


Figure 18: Inverted empirical cumulative probability distribution of the occupant load density for all stores.

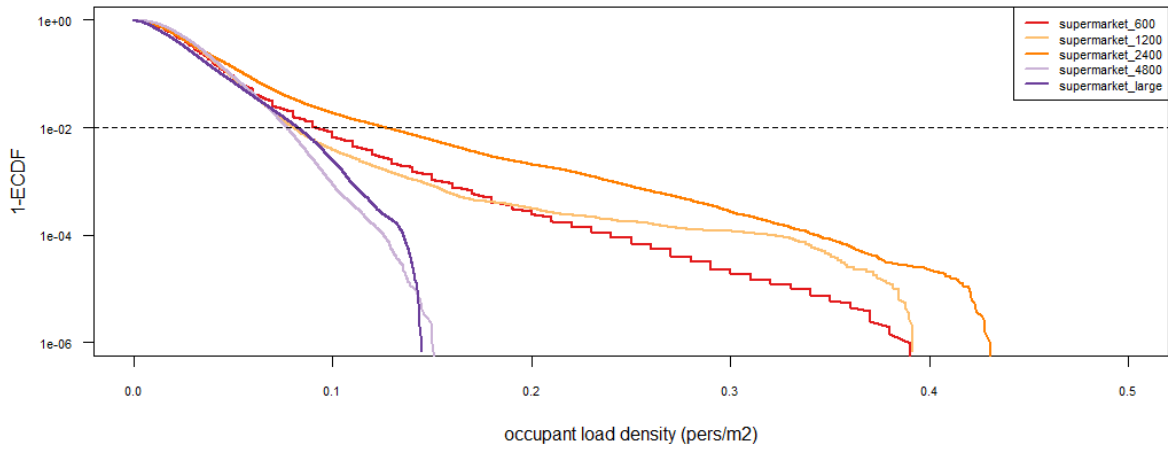


Figure 19: Inverted empirical cumulative probability distribution of the occupant load density for supermarkets (without highly frequented supermarkets).

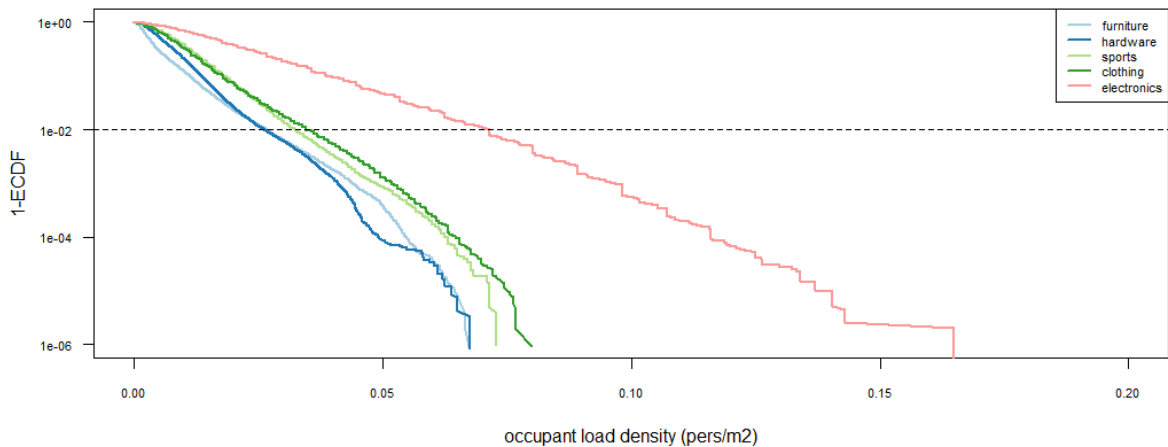


Figure 20: Inverted empirical cumulative probability distribution of the specialist stores.

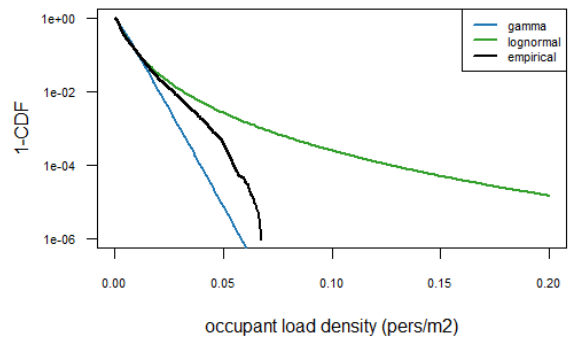
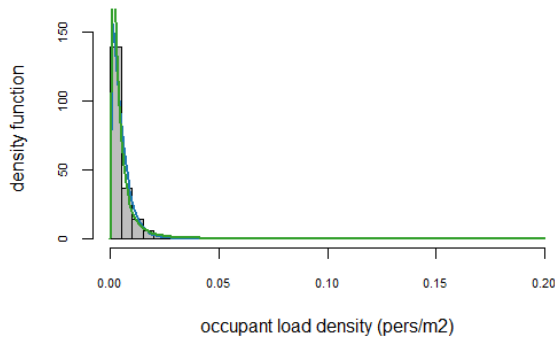
Probabilistic distribution models and distribution parameters

The occupant load density of each retail type is represented by means of a gamma distribution and lognormal distribution. The definition of the distribution can be found in Appendix A3. The estimated parameters are contained in Table 4. Figure 21 shows the comparison between model and data in the histogram (left) and the inverted distribution function (right). Depending on the retail type, either the gamma distribution or the lognormal distribution fits the data better. Most occupant load densities can be represented by a gamma distribution. The lognormal distribution is particularly suitable for retail types where individual stores have a relatively high occupant load density. It should be noted that the goodness of fit for low exceedance probabilities (e.g. $< 10^{-4}$) are prone to statistical uncertainty, since the data often comes from only one or a few stores.

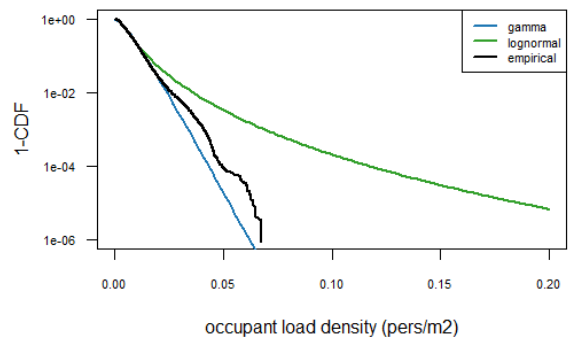
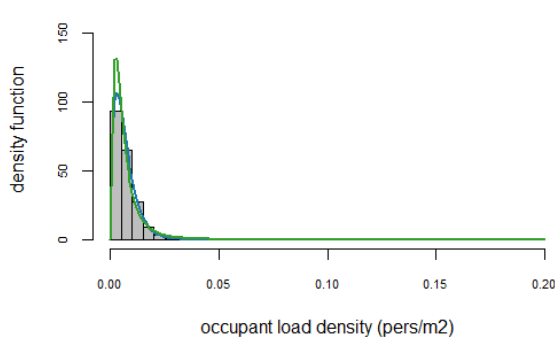
	Gamma distribution		Lognormal distribution		Choice of the best representative distribution
	shape	rate	meanlog	sdlog	
Furnitures	1.22	249.36	-5.78	1.00	Gamma distribution
Hardware (DIY)	1.77	260.07	-5.29	0.85	Gamma distribution
Sports	2.09	217.19	-4.90	0.77	Gamma distribution
Clothing	1.95	210.96	-4.96	0.79	Gamma distribution
Electronics	1.91	95.66	-4.20	0.80	Gamma distribution
Supermarket_600	2.35	89.04	-3.86	0.70	Lognormal distribution
Supermarket_1200	2.29	86.49	-3.87	0.76	Lognormal distribution
Supermarket_2400	1.78	62.24	-3.86	0.83	Lognormal distribution
Supermarket_4800	2.65	96.85	-3.80	0.72	Gamma distribution
Supermarket_large	1.57	71.06	-4.16	0.99	Gamma distribution
Department store	2.08	69.30	-3.76	0.81	Gamma distribution
Mall	1.34	62.24	-4.26	1.04	Gamma distribution
Highly frequented super- markets	1.78	24.23	-2.92	0.84	Gamma distribution

Table 4. Parameters of the gamma and the lognormal distribution for the occupant load density per retail type. The parameters of the distribution that is not representative are greyed.

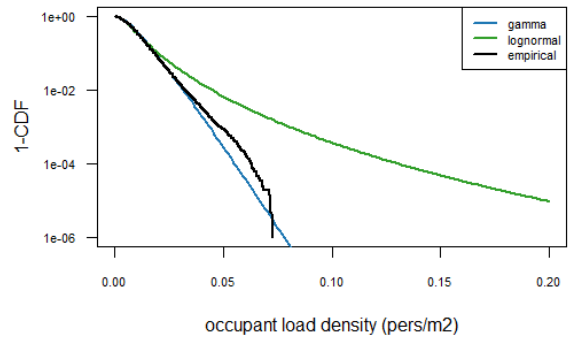
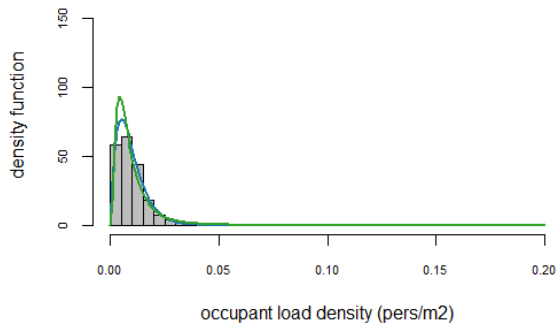
furniture



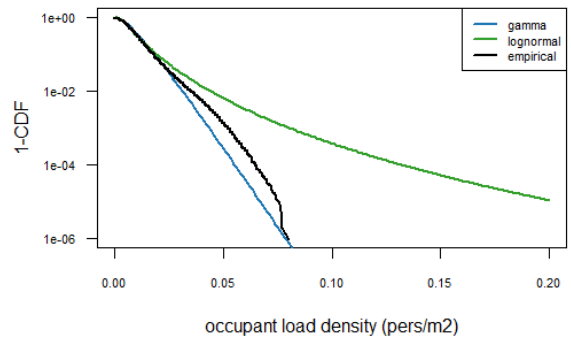
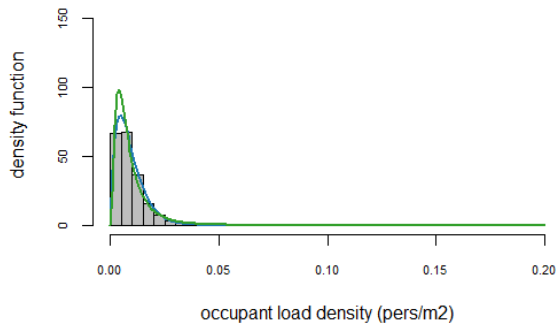
hardware



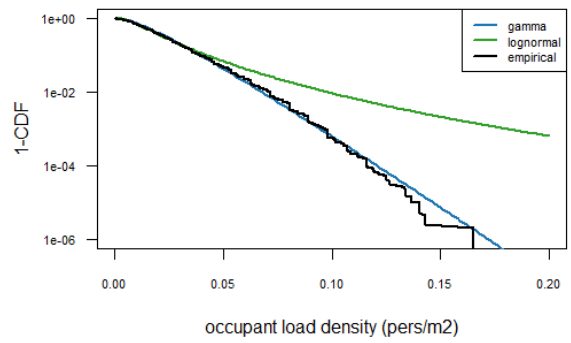
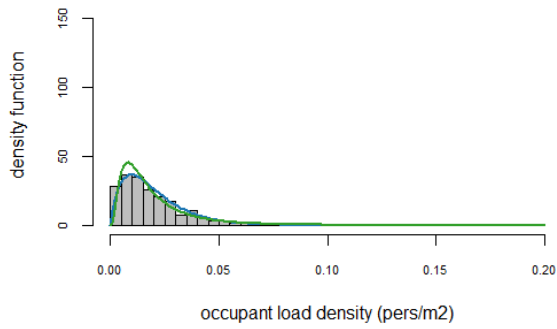
sports



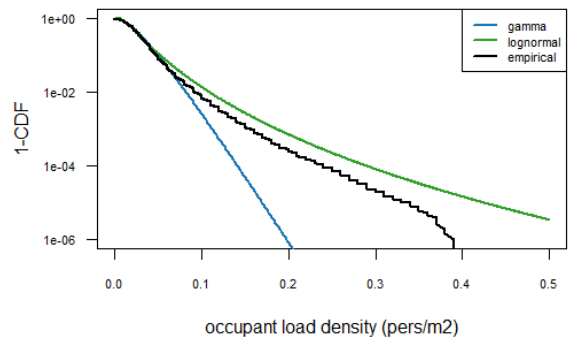
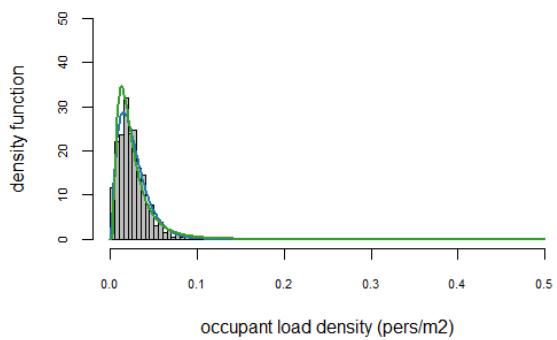
clothing



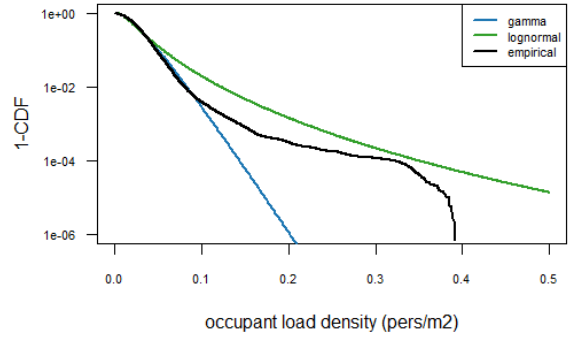
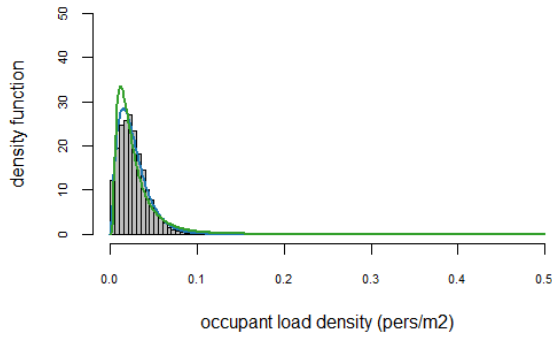
electronics



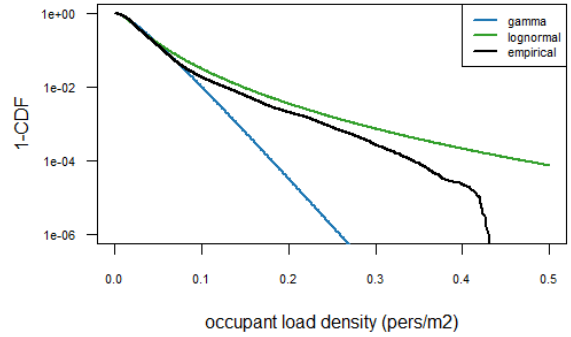
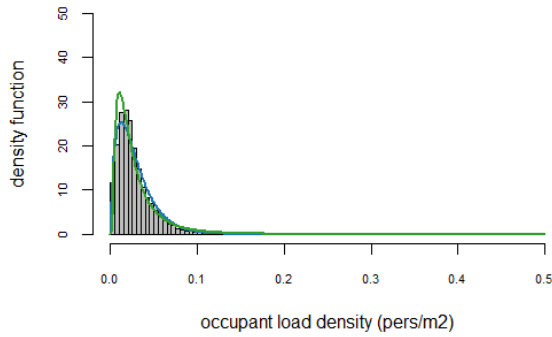
supermarket_600



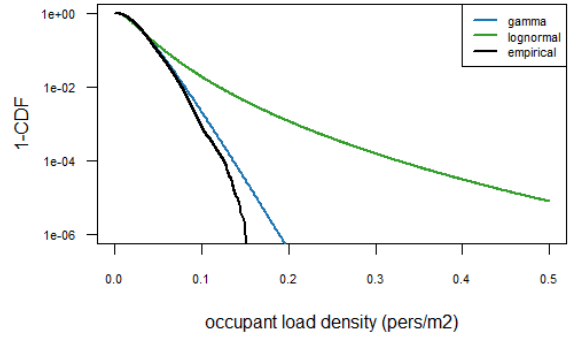
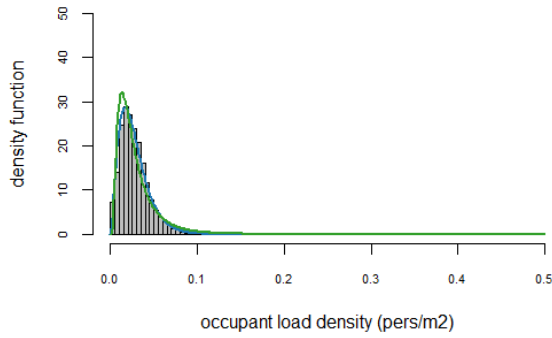
supermarket_1200



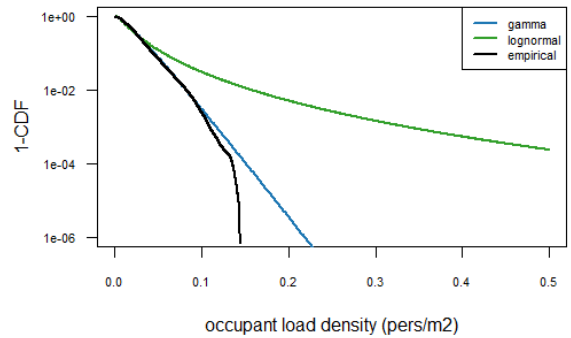
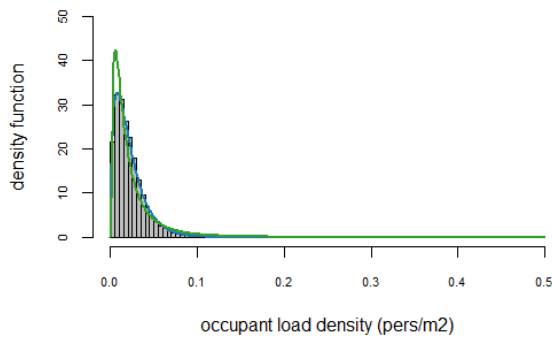
supermarket_2400



supermarket_4800



supermarket_large



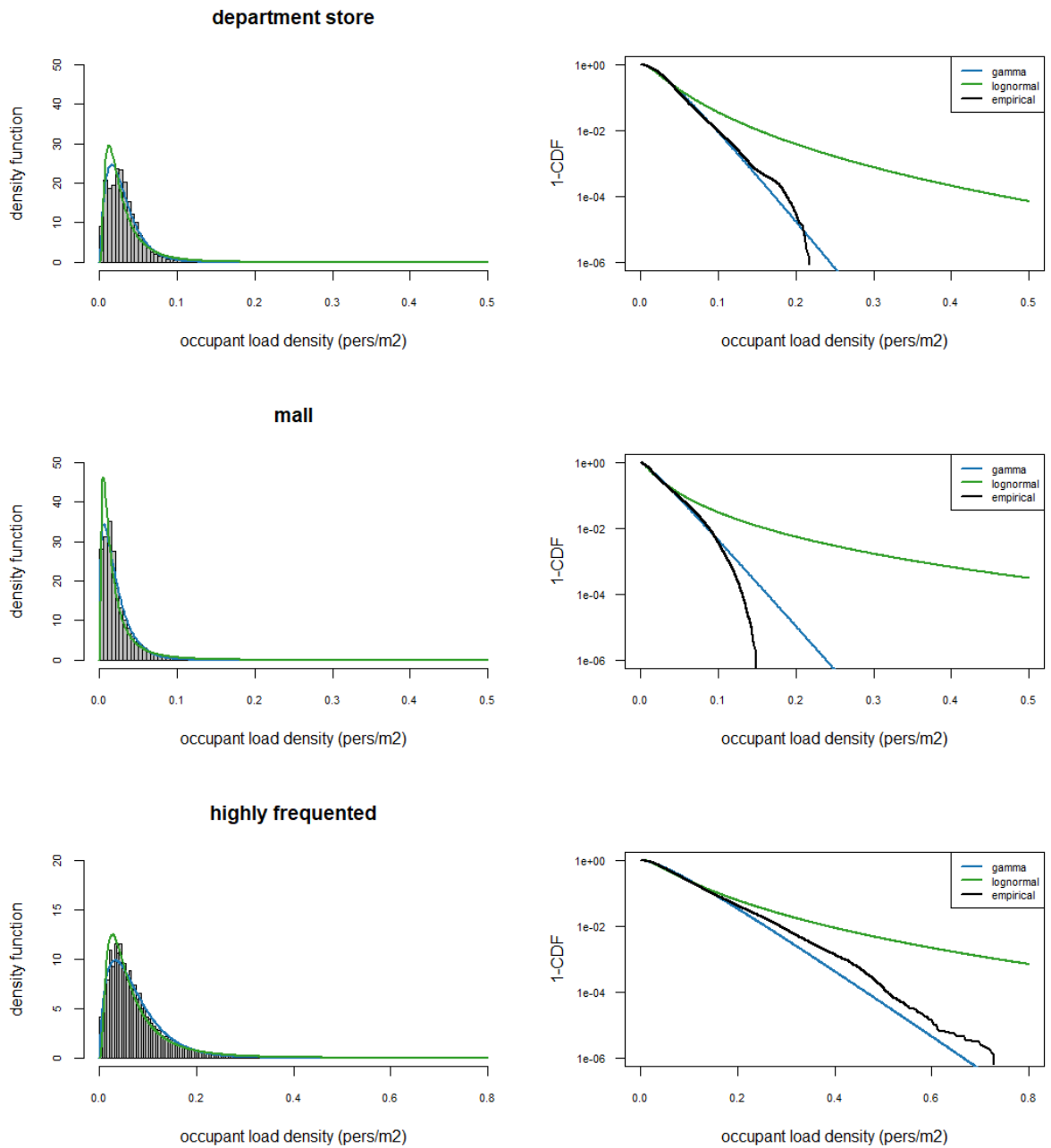


Figure 21. Comparison of the probabilistic model and the data as histogram (left) and as inverted cumulative probability distribution.

Retail type	Mean value	standard deviation	Coefficient of variation	Median	80%-Quantile	90%-Quantile	99%-Quantile	Max. measured value
Furnitures	0.005	0.005	1.110	0.003	0.007	0.011	0.027	0.068
Hardware (DIY)	0.007	0.005	0.802	0.005	0.010	0.014	0.026	0.068
Sports	0.009	0.007	0.742	0.008	0.014	0.018	0.033	0.073
Clothing	0.009	0.007	0.810	0.007	0.014	0.018	0.035	0.081
Electronics	0.019	0.015	0.791	0.015	0.029	0.039	0.071	0.165
Supermarket_600	0.025	0.019	0.754	0.022	0.038	0.048	0.091	0.400
Supermarket_1200	0.026	0.018	0.675	0.023	0.038	0.048	0.080	0.396
Supermarket_2400	0.029	0.025	0.874	0.022	0.041	0.055	0.126	0.438
Supermarket_4800	0.027	0.016	0.586	0.025	0.039	0.048	0.077	0.151
Supermarket_large	0.022	0.017	0.791	0.018	0.033	0.044	0.082	0.145
Department store	0.030	0.020	0.679	0.027	0.044	0.055	0.099	0.218
Mall	0.022	0.019	0.879	0.016	0.033	0.048	0.088	0.150
Highly frequented supermarket	0.073	0.059	0.807	0.057	0.107	0.147	0.286	0.743

Table 5: Compilation of sample statistics of the occupant load density in pers./m².

4.3.3 Floor level

In some fire protection standards, the occupant load density is distinguished between the floor levels. For this purpose, the quantiles of the occupant load density per retail type are evaluated according to the floor level (see Figure 22). The stores located on the main floor tend to have a slightly lower occupant load density. This contradicts some of the definitions in the standards (see Figure 2). However, a clear tendency over all retail types is not observable. Note that the result should be treated with caution due to the small sample size (see Table 6). Nevertheless, the interpretation often found in fire protection standards, which assumes a reduction in the occupant load density for basements and upper floors, is therefore questionable. Highly frequented stores, e.g. in cities, are usually strategically placed. For these buildings the floor level of the store is less important for customers.

Retail type	main floor	basement / upper floors
Furnitures	6	0
Clothing	2	0
Mall	5	3
Electronics	6	4
Highly frequented supermarkets	3	2
Department store	0	4
Furnitures	3	2
Sports	2	3
Supermarket_1200	6	0
Supermarket_2400	7	2
Supermarket_4800	6	2
Supermarket_600	9	0
Supermarkt_large	5	2

Table 6: Number of samples for the statistical evaluation per floor level and per retail type.

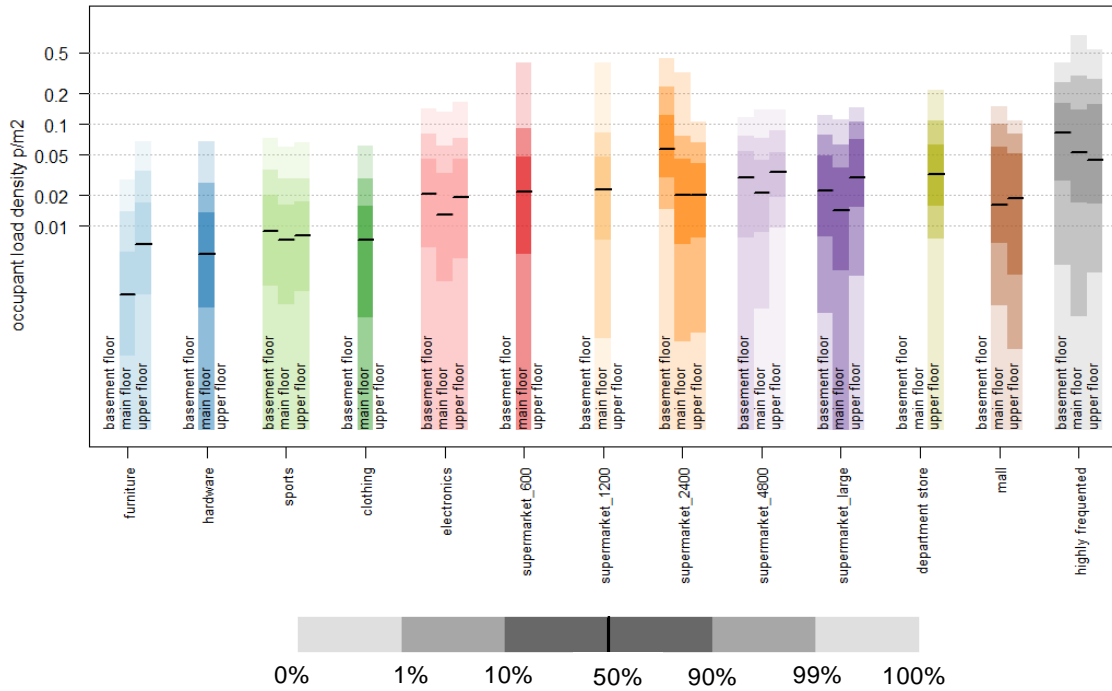


Figure 22: Quantiles of the occupant load density per retail type and per floor level (basement floor / main floor / upper floor). The occupant load density is visualised on a logarithmic scale for reasons of illustration.

4.3.4 Quantiles under consideration for the measurement error

Figure 23 shows the 99% quantile of the occupant load density of all measured stores $\pm \Delta$ as a function of the daily visitor frequency per square meter. The measurement error of the assessed occupant load density is generally greater with high visitor frequency. Therefore, the measurement error has the largest influence on the highly frequented supermarkets. The measurement of the store with the highest visitor frequency of 16 pers./m² per day (highly frequented supermarket) also shows the highest measurement error of $\Delta = 0.20$ pers./m², with a 99% quantile of 0.26 pers./m².

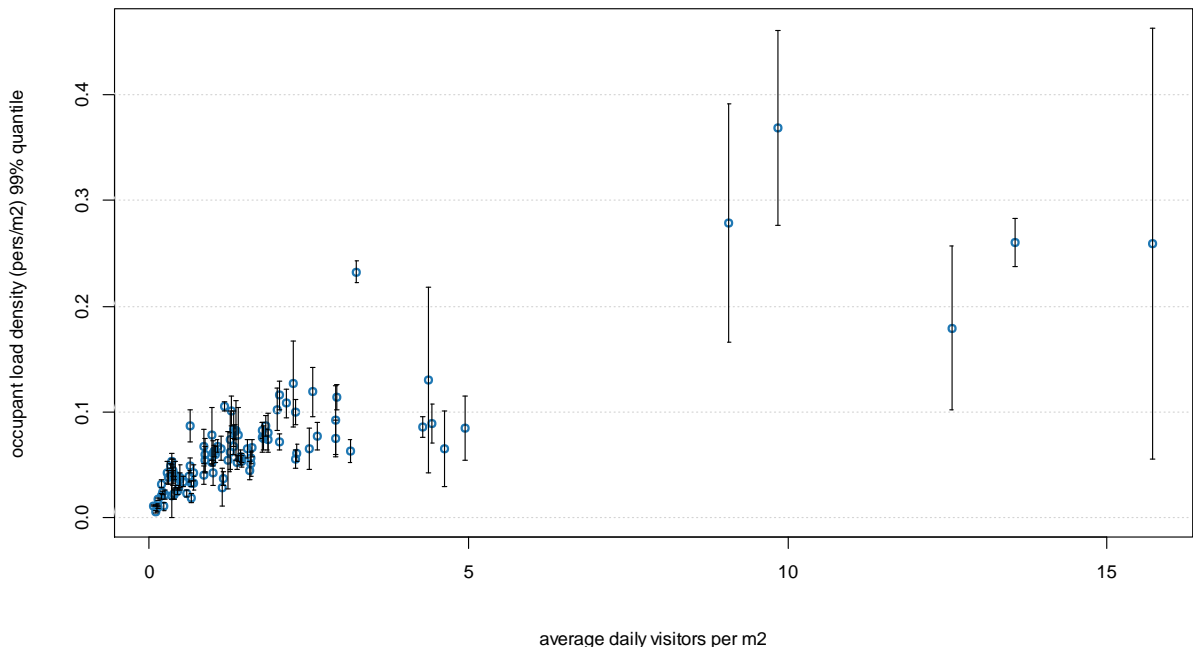


Figure 23: 99% quantile of the occupant load density of all measured stores $\pm\Delta$ as a function of the daily visitor frequency per square meter.

4.3.5 Maximal occupant load density per day

Chapter 2.4 discusses the correlation between occupant load and the fire ignition rate. It is argued that a weak correlation or even an independence can be expected and that this justifies the evaluation based on minute-per-minute steps. The other extreme would be a complete correlation of occupancy with fire ignition rate, i.e. fires take place when there is a high occupant load. As an extreme value assessment, therefore, the occupant load density at the daily maximum is evaluated for each retail type (Figure 24) assuming the fire starts at peak times of the day.

Compared to the evaluation of all data (Figure 17), the evaluation of the daily maximum has higher values for the lower quantiles. For the upper quantiles, however, the influence is only small, since these values are often already determined by the daily maxima.

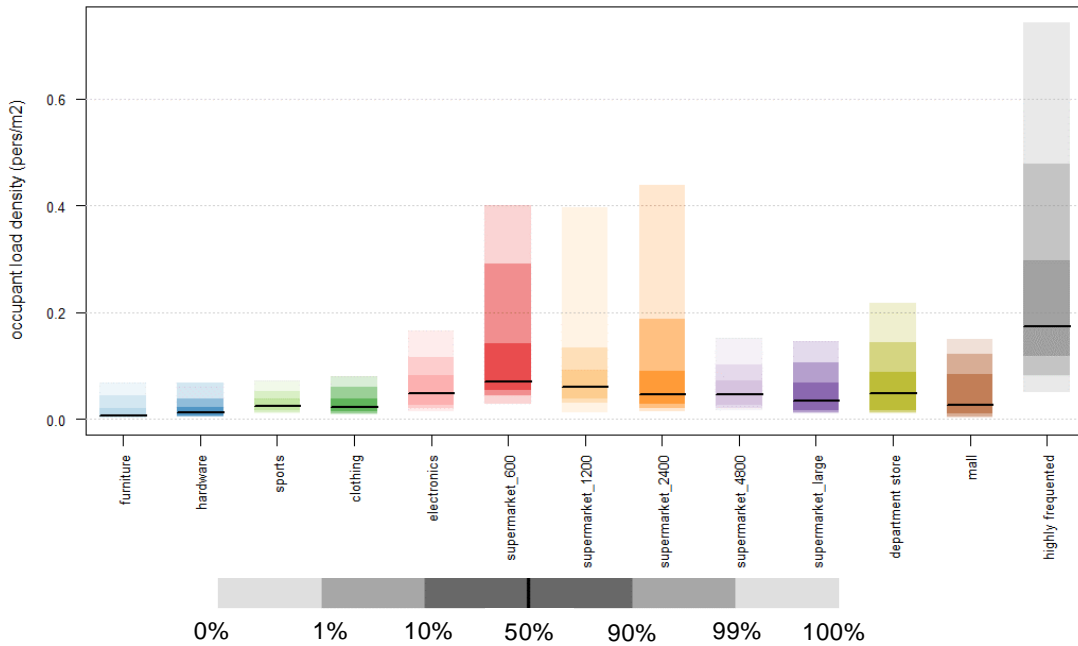


Figure 24: Quantiles of the daily maximum of the occupant load density per user category.

4.3.6 Sales area

Figure 25 to Figure 27 show the effect of the sales area on the occupant load density. There is no clear identifiable dependency. In Figure 25 there is a decrease of the occupant load but mainly driven by the retail type. An area-related gradation of the occupant load density is therefore not suitable.

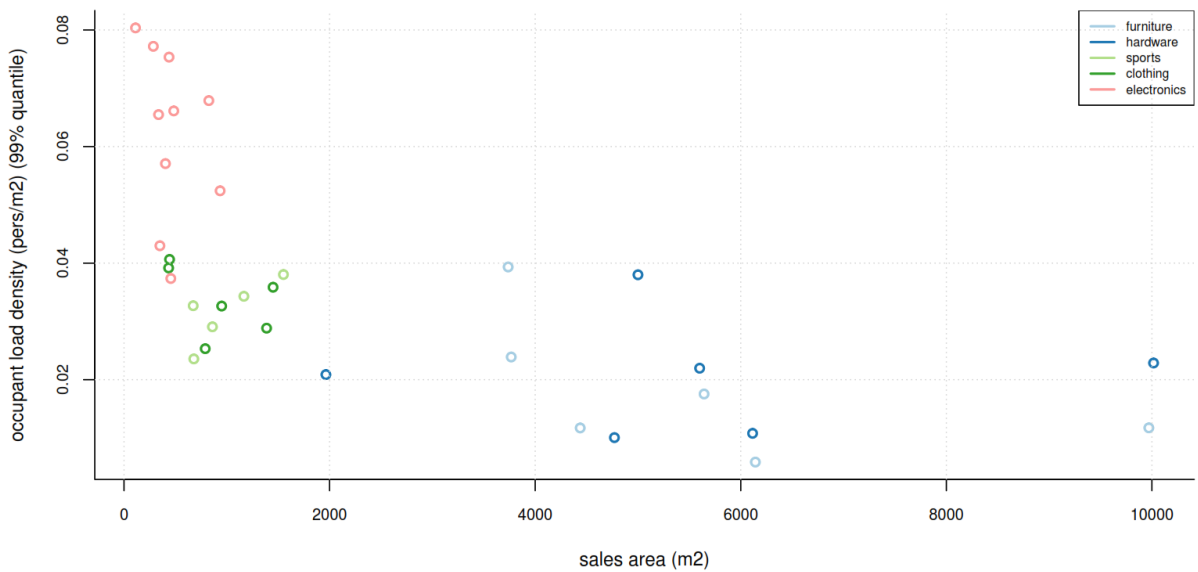


Figure 25: Effect of the sales area on the occupant load density (99% quantile) for non-food stores.

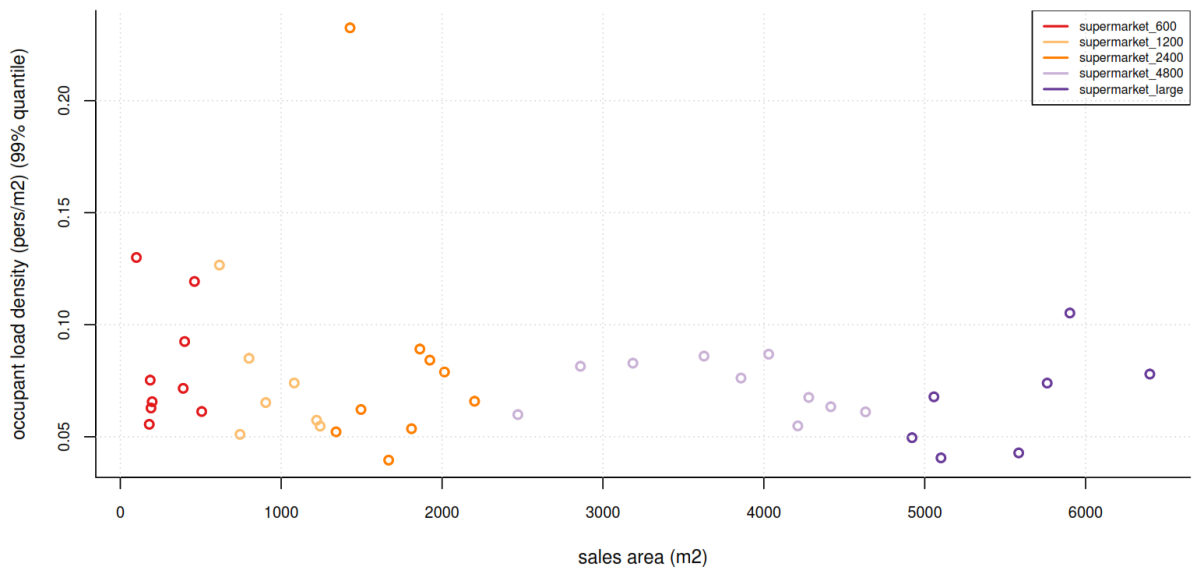


Figure 26: Effect of the sales area on the occupant load density (99% quantile) for supermarkets.

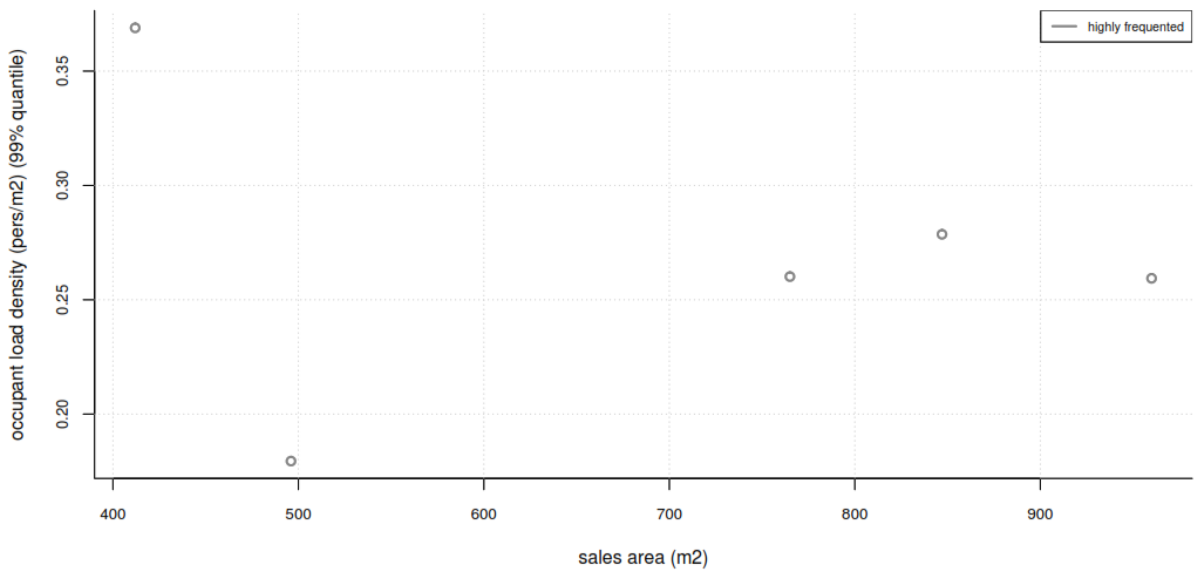


Figure 27: Effect of the sales area on the occupant load density (99% quantile) for highly frequented supermarkets.

4.3.7 Dwell Time

The dwell time is not measurable with the measurement system in use. However, the average dwell time can be estimated by Little's Law [15]. To estimate the average dwell time, the system is represented as a queuing system. There is a specific number of persons that arrive, wait in the system, and then leave the system. According to Little's Law, under steady state conditions, the average number of persons in a queuing system $E[P_{Little}]$ equals the average rate λ at which the persons arrive multiplied by the average time that a person spend in the system $E[W]$:

$$E[P_{Little}] = \lambda \cdot E[W] = \frac{p_{WD}}{h_{WD}} \cdot E[T]$$

The average time that a person spends in the system $E[W]$ can be seen as the average dwell time $E[T]$ illustrated in Figure 28 and the average rate λ can be considered as the total number of persons per day p_{WD} divided by the opening hours h_{WD} . According to Little [15] this law is

remarkably general and simple because it does not require any information about the number of checkout counters or the underlying distribution of the dwell time. However, under real conditions, the steady state condition required for this law is violated.

The distribution of the average dwell time is illustrated in Figure 28. Malls have the highest average dwell times. However, there is a large spread of the values. The shortest average dwell time is associated with highly frequented supermarkets. For normal supermarkets, the average dwell time increases with increasing area. This indicates that large stores are visited for larger purchases that take longer. Somewhat surprisingly, a relatively short time is spent in furniture stores. It must however be noted that the data collection concerns normal furniture stores and not IKEA stores where people stay longer due to a customer experience.

The average occupant load density can be estimated by the average dwell time and data on the total number of arrivals or departures of a store. This data can be obtained by counting systems or by timestamp of cash slips. Especially data for the latter is largely available or easy to assess. Assuming that the distribution of the occupant load density is similar to those in Switzerland (e.g. the same coefficient of variation as in Table 5), the complete distribution density can be estimated.

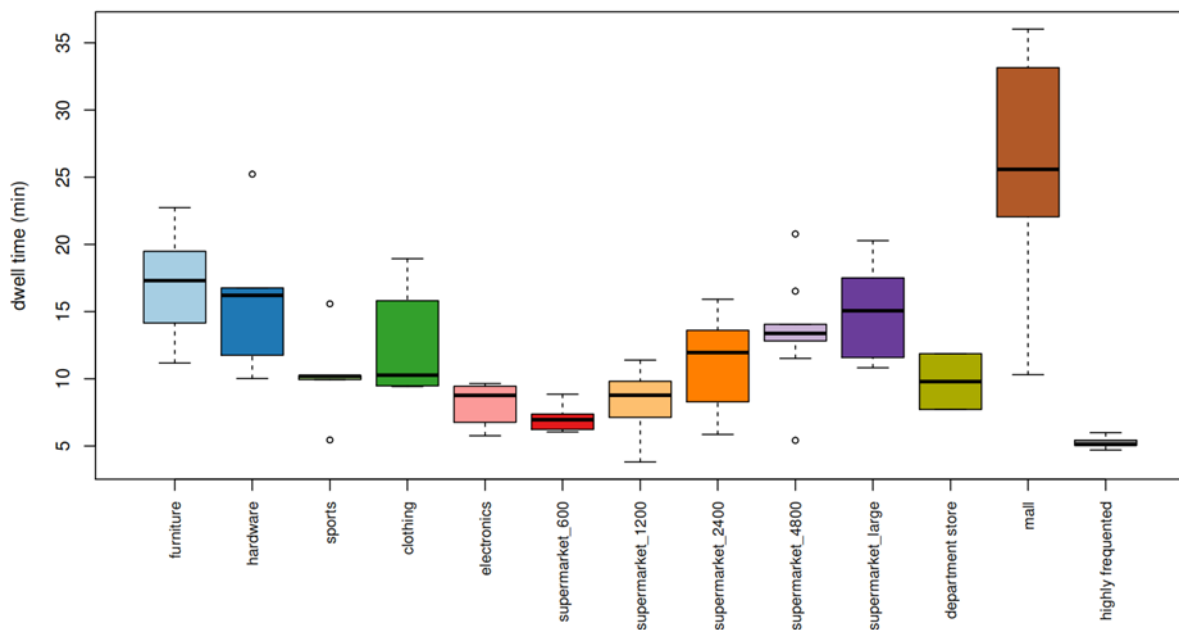


Figure 28: Average dwell time.

4.4 Limitations on the use of the data

The data on the occupant load density in retail buildings is, strictly speaking, only valid for Switzerland. However, the results may be used in countries with similar conditions to Switzerland in the case of:

- A similar spatial distribution of stores and a similar distribution of the population in urban and rural areas (not necessary for the use of the data for highly frequented supermarkets)
- Comparable opening hours, e.g. usually 10 – 12 hours per day, no Sunday selling (except for highly frequented supermarkets)

— Similar cultural shopping behaviour, e.g. shopping peaks at noon and in the evening. Big purchases on Saturdays in large supermarkets (influences dwell time).

Note that customers' needs will be similar across countries: no long waiting times at checkouts, similar product portfolios, similar coverage areas in cities and rural areas. Therefore, it can be expected that the occupant load densities will not deviate in an extreme manner from the Swiss data. Further, it can be expected that the variability of the data is observable across countries and that the distribution has a large tail.

5. Remarks on the determination of design values

Based on the results of the study, design values can be derived for the occupant load density in retail stores. This chapter discusses what needs to be considered.

Retail type

The statistical survey revealed that the density of persons depends strongly on the retail type. Therefore, it is proposed that design values for the occupant load density should be differentiated according to three retail categories:

- **Non-food, (near-food) stores**, department stores and shopping centres: specialist stores with no or only a small proportion of food items. The retail type includes specialist stores such as furniture stores, DIY stores, sporting goods, clothing, and electronics. Near-food stores (e.g. personal care, pet supplies) also fall into this category as well as areas in department stores and shopping centres with no or only a small proportion of food items. Note that the distribution of the individual categories varies considerably, however, only within small occupant load densities, e.g. less than 0.1 pers./m². Therefore, it is reasonable to put these categories together in one overarching category.
- **Supermarkets**: stores or areas that mainly sell food items but can also have near-food and non-food items in their assortment.
- **High-frequency supermarkets**: These stores or areas include supermarkets at major transport hubs with Sunday sales. As a rule of thumb, a total daily number of visitors of more than 7 persons per square metre of gross floor area per day can be expected.

Floor level

The influence of the floor level on the occupant load density across fire safety guidelines is not clear and becomes evident when comparing different normative interpretations (see Figure 2). The statistical analysis showed that, in contrast to the current interpretation present in the guidelines of many countries, a higher occupant load density can be expected on basement levels and the upper floor levels. However, the occupancy rate is strongly dependent on the location of the store (rural or urban) and the products offered. If there is a certain demand for a product, the floor on which it is offered plays only a small role.

It is to be expected that in many standards the determination of the occupant load density for basement and upper floor levels is associated with setting higher requirements for the means of egress, e.g. larger exit widths. The reason for higher requirements is usually a higher risk perception for these floor levels. For example, an increase in the number of persons can result in wider exit widths. Stricter requirements for multi-storey stores are legitimate. However, these should be achieved for example by means of a floor-wise differentiated requirement for the necessary width per occupant of exits and not by an artificial increase of the surveyed occupant load density.

The number of floors affects the occupant load density if additional products are offered along the walkway to other floor levels. This is the case, for example, in department stores where products are strategically placed, usually on the main floor. In specialist stores and supermarkets, such strategic product placement plays a subordinate role regarding the distribution of the occupant load. For department stores and malls, a floor-by-floor differentiation of the occupant load density makes sense. However, this differentiation depends strongly on the sales strategy. For this reason, it is recommended that, when designing the means of egress for department stores and malls, first, the total occupant load of the building should be determined

according to different retail types. In a second step the occupant load can be distributed over the floors by considering the selling strategy.

For this reason, it is recommended not to differentiate the occupant load density in a generalised way according to floor level.

Size of the selling area

The survey for supermarkets included five different selling area categories. No clear correlation between sales area and occupant load density could be observed. For this reason, no area-related gradation of the occupant load density is recommended.

Quantile for the design value

The statistical analysis of the occupant load density revealed that it is very variable. In performance-based design guidelines a worst-credible case is usually required for determining a parameter. However, it is rarely further described how such a case is actually defined. This makes it difficult to define a quantile in order to derive a design value for the occupant load density.

An appropriate choice for a design value could be derived by introducing a semi-probabilistic design approach for evacuation problems. In semi-probabilistic design approaches, safety factors are introduced for key-parameters of the code format. These safety factors are calibrated in order to reach a specified risk-based acceptance criterion. The semi-probabilistic design concept has found application in the design of structures in fire [16,17]. For evacuation problems, however, the state of research has not gone beyond a conceptual phase of this approach [18]. The reason was that risk-based acceptance criteria for evacuation problems were lacking and the limited availability of probabilistic models. This gap is closed by the research performed at ETH Zurich [19,20] where risk-based acceptance criteria were defined and risk-based optimisations of systems have been performed.

As a starting point – without performing any risk analysis – a reasonable choice for a design value is a higher quantile value. Therefore, a 99% quantile value is selected as the tentative design value for the occupant load density. This value will not be exceeded in 99% of cases within the opening hours of a store.

In comparison to other safety areas, the choice of a 99% quantile value is more likely to be rated as high in comparison to other safety fields. For example, for structural fire design an 80% quantile value for the fire load is taken as basis for the design value.

In Table 7 the 99% quantiles are rounded up to determine the design values. This supports the user-friendliness of the design values and covers uncertainties in the survey.

	Non-Food [pers./m ²]	Supermarket [pers./m ²]	Highly frequented supermarket [pers./m ²]
99%-Quantile	0.099	0.126	0.286
Tentative design value	0.10	0.15	0.30

Table 7: Tentative design values for the occupant load density in retail stores.

Application of the values

The values may not be used for prescriptive guidelines without further investigation into the effects of a changing these values, as the occupant load density directly influences the dimensioning of escape route requirements. This can have a significant impact on the risk to life. For an adjustment of these values, an investigation using probabilistic and risk-based methods is recommended. The probabilistic models of the occupant load density from this study (see Table 4) can be used for this purpose.

For performance-based approaches the values in Table 7 can be used, keeping in mind that it corresponds to a 99% quantile and it is not clear whether it corresponds to a worst-credible case. However, when considering the large variability of the occupant load density, the question arises whether it is justified to take a high occupant load when analysing rare fire scenarios, e.g. related with a small occurrence rate or a high value for the fire growth rate and other fire related parameters. Only a probabilistic or a risk-based analysis can answer this question.

Regarding the large variability of the occupant load, the question also arises whether it is justified to apply complex models for safety evaluation, e.g. based on agent-based evacuation analysis, in comparison to more simplified models, e.g. based on the hydraulic analogy. This should be discussed in future research by performing probabilistic analysis with the consideration of all major uncertainties and assessing the importance of each uncertainty on the output. This provides the basis for deriving design evacuation scenarios, e.g. by a semi-probabilistic design approach.

When probabilistic or risk-based approaches are chosen within a performance-based design it is recommended to use the probabilistic distribution specified in Table 4. The advantage of using such an approach is that the skewness of the distribution of the occupant load density can be considered.

6. Conclusions

A data survey was conducted in Switzerland on the occupant load density in 96 retail stores over a measurement period of one year. Supermarkets, non-food stores (including department stores and malls), as well as supermarkets with a high-frequency use were investigated. The densities show considerable differences for the design values in several codes and guidelines. Therefore, across several countries, the values from fire protection standards and guidelines may be set too high overall. However, for highly frequented supermarkets at important public traffic hubs, the values might be set too low according to some standards.

A store's retail type has a big impact on the occupant load density and should be considered when setting design values for fire protection standards and guidelines. We propose a differentiation between food stores (supermarkets), non-food stores (including department stores and malls), and highly frequented supermarkets located at important public traffic hubs.

In some current fire protection standards and guidelines, the occupant load density depends on the floor level. The data does not provide an obvious trend that would justify assigning either more or less persons to a specific floor level. This contradicts the current implementation in standards and guidelines. It should be noted that the occupant load depends mainly on the strategic allocation of the store location and its products and not necessarily on the floor level.

The data shows a high variability of the occupant load density and the distribution is heavily right skewed. Therefore, the simultaneity of a high occupant load density and a (rare) severe fire is rated as non-credible. This should be considered in the design of the means of egress when applying performance-based design approaches. An appropriate consideration of the distribution of the occupant load is only possible when applying probabilistic or risk-based approaches. The role of the variability of the occupant load density in retail stores in relation to other uncertainties in evacuation analysis and in fire situations should be the focus of future research.

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A1 List of stores participating in the survey

Organisation	Object	Format	Retail type	Floor level
Migros	Marin Centre	Mall	Mall	Main floor
Migros	Marin Centre	Mall	Mall	Upper floor
Migros	Marin Centre	Super- und Verbrauchermarkt	Supermarket_large	1. Basement
Migros	Portes-Rouges	Super- und Verbrauchermarkt	Supermarket_1200	Main floor
Migros	Bern - Westside	Melectronics	Electronics	1.Basement
Migros	Interlaken	Melectronics	Electronics	1.Upper floor
Migros	Schönbühl - Shopyland	Melectronics	Electronics	1.Upper floor
Migros	Thun - Oberland	Melectronics	Electronics	Main floor
Migros	Bern - Welle 7	Melectronics	Electronics	1. Basement
Migros	Brügg	Super- und Verbrauchermarkt	Supermarket_large	Main floor
Migros	Kirchberg	Super- und Verbrauchermarkt	Supermarket_2400	Main floor
Migros	Lostdorf	Super- und Verbrauchermarkt	Supermarket_600	Main floor
Migros	Wynecenter	Super- und Verbrauchermarkt	Supermarket_large	Upper floor
Migros	Bern - Bahnhof	Super- und Verbrauchermarkt	Highly frequented supermarket	1. Upper floor
Migros	Waldstätterstrasse	Super- und Verbrauchermarkt	Supermarket_1200	Main floor + 1. Upper floor
Migros	Bülach - Süd	Mall	Mall	Building
Migros	Bülach - Süd Fachmarkt	SportXX	Sports	1. Basement
Migros	Bülach - Süd Fachmarkt	Do-it Garden	Hardware (DIY)	Main floor
Migros	Volketswil - MParc	Mall	Mall	Main floor
Migros	Volketswil - MParc	Mall	Mall	1.Upper floor
Migros	Volketswil - MParc	Micasa	Furniture	1.Upper floor
Migros	Volketswil - MParc	Obi	Hardware (DIY)	Main floor
Migros	Wädenswil	Melectronics	Electronics	Main floor
Migros	Wädenswil - Zürisee Center	SportXX	Sports	Main floor
Migros	Zürich - Altstetten	Mall	Mall	Main floor
Migros	Zürich - Altstetten	Mall	Mall	1.Upper floor
Migros	Zürich - Altstetten	Super- und Verbrauchermarkt	Supermarket_4800	Main floor + 1.Upper floor
Migros	Zürich - Wiedikon M-Märt	Mall	Mall	Building
Migros	Zürich - M-City	Mall	Mall	Building
Migros	Zürich - M-City	Super- und Verbrauchermarkt	Supermarket_4800	2.Upper floor+3.Upper floor
Migros	Gossau	Super- und Verbrauchermarkt	Supermarket_2400	Main floor
Migros	Limmatplatz	Super- und Verbrauchermarkt	Supermarket_4800	1.Basement
Migros	Zürich HB	Super- und Verbrauchermarkt	Highly frequented supermarket	Basement
Migros	Illuster	Super- und Verbrauchermarkt	Supermarket_large	Main floor
Migros	Zumikon Provisorium	Super- und Verbrauchermarkt	Supermarket_600	Main floor
Migros	Rapperswil	Mall	Mall	Building
Migros	Tourbillon Center	Super- und Verbrauchermarkt	Supermarket_2400	Main floor
Migros	Chablais Centre Aigle (offen)	Mall	Mall	Building
Migros	Genève Aeroport Cointrin	Super- und Verbrauchermarkt	Highly frequented supermarket	Main floor
Migros	Claramarkt	SportXX	Sports	3.Upper floor

Organisation	Object	Format	Retail type	Floor level
Migros	Paradies	SportXX	Sports	Main floor
Migros	Schönthal	SportXX	Sports	1.Basement
Migros	AMRP Amriswil Provisorium	Super- und Verbrauchermarkt	Supermarket_1200	Main floor
Migros	MEPP Mels Pizolpark	Super- und Verbrauchermarkt	Supermarket_4800	Main floor + 1.Upper floor
Migros	WTRO Winterthur Rosenberg	Super- und Verbrauchermarkt	Supermarket_4800	Main floor
Migros	SGOB Micasa St. Gallen	Micasa	Furniture	1.Upper floor
LiB AG	G101 Globus Bahnhofstrasse	Super- und Verbrauchermarkt	Supermarket_2400	1.Basement
LiB AG	G101 Globus Bahnhofstrasse	Kaufhaus	Department store	Upper floor1
LiB AG	G101 Globus Bahnhofstrasse	Kaufhaus	Department store	Upper floor2
LiB AG	G101 Globus Bahnhofstrasse	Kaufhaus	Department store	Upper floor3
LiB AG	G101 Globus Bahnhofstrasse	Kaufhaus	Department store	Upper floor4
LiB AG	Herren Globus Zürich	Mode	Clothes	Building
LiB AG	Herren Globus Baden	Mode	Clothes	Building
LiB AG	Herren Globus Aarau	Mode	Clothes	Building
LiB AG	Globus Falkenplatz	Mode	Clothes	Building
LiB AG	Globus Conthey	Mode	Clothes	Main floor
LiB AG	Globus Schönbühl	Mode	Clothes	Main floor
Coop	Bülach	Bau+Hobby	Hardware (DIY)	Main floor
Coop	Volketswil G-Center	Bau+Hobby	Hardware (DIY)	Main floor
Coop	Biberist	Coop	Supermarket_4800	Main floor
Coop	Conthey	Coop	Supermarket_4800	Main floor
Coop	Dietikon Silber	Coop	Supermarket_4800	Main floor
Coop	Flims	Coop	Supermarket_1200	Main floor
Coop	Giubiasco	Coop	Supermarket_1200	Main floor
Coop	Haag	Coop	Supermarket_4800	Main floor
Coop	Langnau Ilfiscenter	Coop	Supermarket_2400	upper floor
Coop	Matten	Coop	Supermarket_600	Main floor
Coop	Möhl	Coop	Supermarket_2400	Main floor
Coop	Münchenstein Gartenstadt	Coop	Supermarket_4800	Main floor
Coop	Oron-la-Ville	Coop	Supermarket_2400	Main floor
Coop	Porrentruy Ajoie	Coop	Supermarket_2400	Main floor
Coop	St.-Blais	Coop	Supermarket_600	Main floor
Coop	Zuchwil	Coop	Supermarket_1200	Main floor
Coop	Zürich Albisriederplatz	Coop	Supermarket_1200	Main floor
Coop	Bern City Marktgasse	Coop City	Department store	Building/Main floor/1.Upper floor
Coop	Zürich City Oerlikon	Coop City	Department store	Main floor + 1.Basement + 2.Basement
Coop	Haag Center	EKZ	Mall	Main floor + 1.Upper floor
Coop	Heimberg Center	EKZ	Mall	Main floor + 1.Upper floor
Coop	Littoral Centre	EKZ	Mall	Main floor + 1.Upper floor
Coop	Lyssach Center	EKZ	Mall	Main floor + 1.Upper floor + 2.Basement + 1.Basement + Main floor
Coop	Seewen Markt	EKZ	Mall	Main floor + 1.Upper floor
Coop	Kriens	Interdiscount	Electronics	Main floor
Coop	Lyssach	Interdiscount	Electronics	Main floor
Coop	Spreitenbach Shoppi	Interdiscount	Electronics	Main floor
Coop	Winterthur Rosenberg	Interdiscount	Electronics	Main floor

Organisation	Object	Format	Retail type	Floor level
Coop	Dietlikon	Toptip	Furniture	Main floor + 1. Upper floor
Coop	Oberentfelden	Toptip	Furniture	Main floor
Coop	Pratteln	Toptip	Furniture	Main floor
Coop	Canobbio Ipermercato Resega	Coop	Supermarket_large	Main floor
Coop	Bachenbülach	Coop	Supermarket_large	Main floor
Coop	Vernier Blandonnet	Coop	Supermarket_large	Main floor
Coop	Luzern Bahnhof	Coop	Highly frequented supermarket	Main floor
Coop	Aarau Bahnhof	Coop	Highly frequented supermarket	Main floor
Coop	Baden Bahnhof	Coop	Supermarket_2400	Main floor
Volg	Aadorf	Super- und Verbrauchermarkt	Supermarket_600	Main floor
Volg	Fülenbach	Super- und Verbrauchermarkt	Supermarket_600	Main floor
Volg	Koppigen	Super- und Verbrauchermarkt	Supermarket_600	Main floor
Volg	Scheuren Forch	Super- und Verbrauchermarkt	Supermarket_600	Main floor
Volg	Schiers	Super- und Verbrauchermarkt	Supermarket_600	Main floor
Furniture				
Pfister	Marin Centre		Furniture	Main floor
Maus Frères	Bienne	Jumbo	Hardware (DIY)	Main floor
Maus Frères	Chavannes	Jumbo	Hardware (DIY)	Main floor

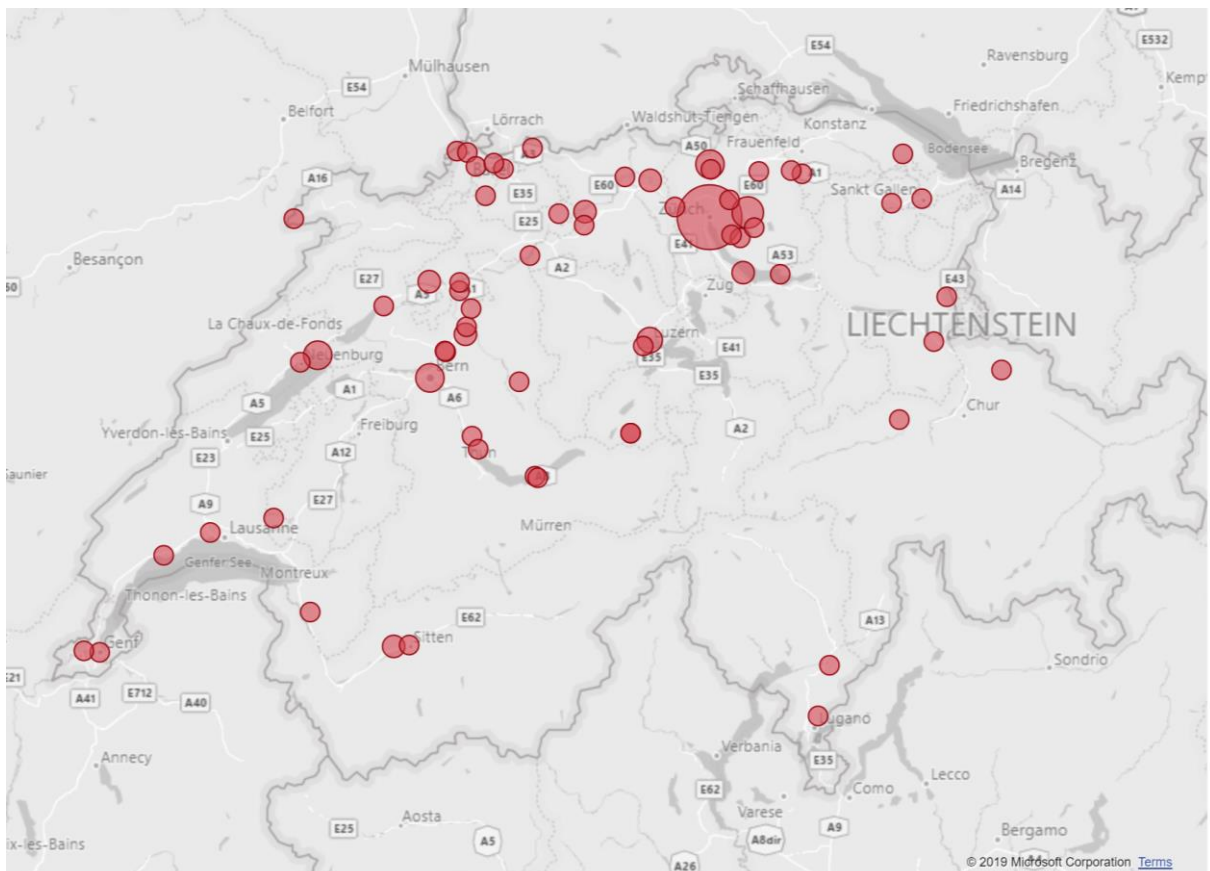


Figure 29: Location of the stores for the survey.

A2 Measurement requirements

A2.1 Documentation

For each object the following information is recorded:

- Retail type
- Floor level
- Sales area
- Opening hours

A2.2 Measurement concept

Measurement system

The occupant load density is determined by measuring the flow of persons at the entrances and exits of the sales areas. For this purpose, optical sensors are used which detect the persons and their direction of movement. Compared to other technologies such as light barriers, radar, or WiFi the use of optical sensors is currently the most precise technology. According to the manufacturer, the measurement error of the optical sensors is less than 2%.

For a measurement with optical sensors, an illuminance of at least 5 lux is required. The sensors must be mounted at a height of at least 220 cm. Depending on the ratio between mounting height and door width, several sensors may be required per counting point. Persons with a height of less than 120 cm are generally not counted. Figure 30 shows an example of a sensor and the measuring principle.

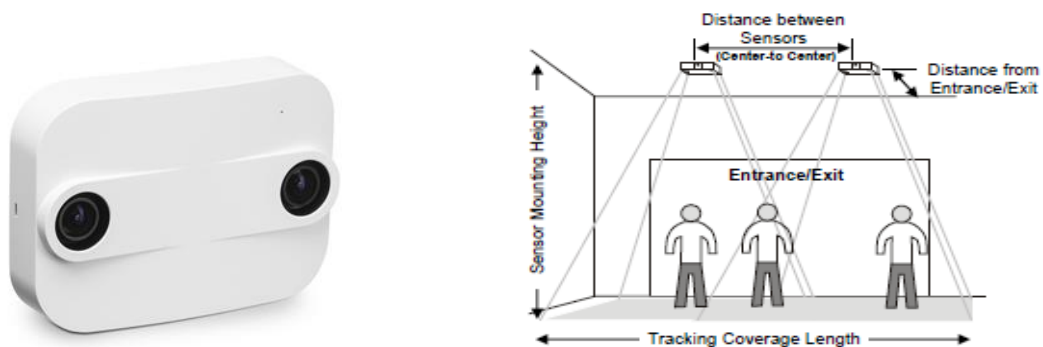


Figure 30 Example of a sensor and measurement principle.

Layout

For each store a layout plan for the surveyed sales areas is requested from the store managers with the following specifications:

- Marking of the sales areas
- All entries and exits to/from the sales area
- Allocation of area usages
- Measuring points including identifiers
- Exits or entries that are not measured must be indicated and explained.

Opening hours and holidays

The occupant load is examined during the regular opening hours per weekday. Holidays are recognised by the very low visitor frequency and are ignored during the evaluation.

Measurement period

A continuous calendar year is recorded per store. The start and end dates differ between the stores. Most of the measurements take place in 2017 and 2018.

Perimeter

All access points to the sales area are measured. Exceptions are access points used by less than 1% of visitors (e.g. elevators). Employee entrances are not recorded.

A2.3 Measurement accuracy

Tolerance

At least 98% accuracy is required for the survey (from 100 recorded persons only a measurement error of ±2 persons is deemed acceptable). This was largely achieved in all stores.

Minimum height of persons

The measuring system only detects persons above a certain height. The requirement placed on the measurement systems was that people must be detected above a height of 120 cm.

High flows of persons

The requirement placed on the measurement system was that the system must deliver accurate results even with high passenger flows of up to 1.2 pers./s per metre access width.

Objects

Only persons are counted. The requirement placed on the measurement system was that other objects, such as shopping carts, should not influence the measurement.

Weather

The requirement placed on the system was that the measurements should not be disturbed by weather influences and that the system must function at any outside temperature (-20°C to 40°C).

Light

The measurement system must function in any daylight and should not be disturbed by external light influences.

A2.4 Data storage

The data is stored in an online database accessible via internet.

Measurement data must be recorded and each direction stored separately at least once a minute. Event-based systems with exact time in seconds are also possible.

The measurement data must be available in an unprocessed form. Pre-calculated person assignments are not permitted.

Example for data storage:

Sensor ID	Time	Count In	Count Out
1065	2017-10-25 11:54:00	10	12
1066	2017-10-25 11:54:00	11	7
1065	2017-10-25 11:53:00	5	3

A3 Probabilistic models

A3.1 Random variables

An upper-case letter X denotes a random variable and a lower case letter x denotes realisation of a random variable X . The probability density function of a random variable X is denoted as $f_X(x)$. The cumulative probability density function of a random variable X is denoted as $F_X(x)$ and its inverse function is denoted as $F_X^{-1}(F_X(x)) = x$.

A3.2 Lognormal distribution

If a random variable X is Lognormal distributed $X \sim \text{LN}(\mu, \sigma)$ then the probability density function $f_X(x)$ is defined through:

$$f_X(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(x) - \mu}{\sigma}\right)^2\right)$$

The parameters μ and σ are the distribution parameters of the Lognormal distribution. The probability density function is defined over $x > 0$. The expected value $E[X]$ (or mean value) of X and the variance $\text{Var}[X]$ of X is assessed by:

$$E[X] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

$$\text{Var}[X] = (\exp(\sigma^2) - 1) \cdot E[X]^2$$

A3.3 Gamma distribution

If a random variable X is Gamma distributed $X \sim \text{GA}(p, b = 1/\text{rate})$ then the probability density function $f_X(x)$ of X is defined according to:

$$f_X(x|a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} \exp\left(\frac{-x}{b}\right)$$

The parameters a and $b = 1/\text{rate}$ are the distribution parameters of the Gamma distribution and $\Gamma(\cdot)$ is the gamma operator. The probability density function is defined over $x > 0$. The expected value $E[X]$ (or mean value) of X and the variance $\text{Var}[X]$ of X is assessed by:

$$E[X] = a \cdot b$$

$$\text{Var}[X] = a \cdot b^2$$